Caribbean Examinations Council

CAPE®
Integrated Mathematics

SYLLABUS
SPECIMEN PAPER
MARK SCHEME

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Integrated Mathematics

Mathematics promotes intellectual development, is utilitarian and applicable to all disciplines. Additionally, its aesthetics and epistemological approaches provide solutions fit for any purpose. Therefore, Mathematics is the essential tool to empower people with the knowledge, competencies and attitudes which are precursors for this dynamic world.

This course is designed for all students pursuing CXC associate degree programme, with special emphasis to those who do not benefit from the existing intermediate courses that cater primarily for mathematics career options. It will provide these students with the knowledge and skills sets required to model practical situations and provide workable solutions in their respective field of study. These skills include critical and creative thinking, problem solving, logical reasoning, modelling ability, team work, decision making, research techniques, information communication and technological competencies for life-long learning. Such holistic development becomes useful for the transition into industry as well as research and further studies required at tertiary levels. Moreover, the attitude and discipline which accompany the study of Mathematics also nurture desirable character qualities.

The Integrated Mathematics Syllabus comprises three Modules:

Module 1 - Foundations of Mathematics
Module 2 - Statistics
Module 3 - Calculus

CARIBBEAN EXAMINATIONS COUNCIL

Caribbean Advanced Proficiency Examination®
CAPE®

INTEGRATED MATHEMATICS SYLLABUS

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Introduction

The Caribbean Advanced Proficiency Examination (CAPE) is designed to provide certification of the academic, vocational and technical achievement of students in the Caribbean who, having completed a minimum of five years of secondary education, wish to further their studies. The examinations address the skills and knowledge acquired by students under a flexible and articulated system where subjects are organised in 1-Unit or 2-Unit courses with each Unit containing three Modules. Subjects examined under CAPE may be studied concurrently or singly.

The Caribbean Examinations Council offers three types of certification at the CAPE level. The first is the award of a certificate showing each CAPE Unit completed. The second is the CAPE Diploma, awarded to candidates who have satisfactorily completed at least six Units, including Caribbean Studies. The third is the CXC Associate Degree, awarded for the satisfactory completion of a prescribed cluster of *eight* CAPE Units including Caribbean Studies, Communication Studies *and Integrated Mathematics. Integrated Mathematics is not a requirement for the CXC Associate Degree in Mathematics.* The complete list of Associate Degrees may be found in the CXC Associate Degree Handbook.

For the CAPE Diploma and the CXC Associate Degree, candidates must complete the cluster of required Units within a maximum period of five years. *To be eligible for a CXC Associate Degree, the educational institution presenting the candidates for the award, must select the Associate Degree of choice at the time of registration at the sitting (year) the candidates are expected to qualify for the award.* Candidates will not be awarded an Associate Degree for which they were not registered.

Integrated Mathematics Syllabus

♦ RATIONALE

The Caribbean society is an integral part of an ever-changing world. The impact of globalisation on most societies encourages this diverse Caribbean region to revisit the education and career opportunities of our current and future citizens. A common denominator is for Caribbean societies to create among its citizens a plethora of quality leadership with the acumen required to make meaningful projections and innovations for further development. Further, learning appropriate problem-solving techniques, inherent to the study of mathematics, is vital for such leaders. Mathematics promotes intellectual development, is utilitarian and applicable to all disciplines. Additionally, its aesthetics and epistemological approaches provide solutions fit for any purpose. Therefore, Mathematics is the essential tool to empower people with the knowledge, competencies and attitudes which are required for academia as well as quality leadership for sustainability in this dynamic world.

The Integrated Mathematics course of study will provide students with the knowledge and skills sets required to model practical situations and provide workable solutions in their respective field of study. These skills include critical and creative thinking, problem solving, logical reasoning, modelling ability, team work, decision making, research techniques, information communication and technological competencies for life-long learning. Such holistic development becomes useful for the transition into industry research and further studies required at tertiary levels. Moreover, the attitude and discipline which accompany the study of Mathematics also nurture desirable character qualities.

This syllabus will contribute to the development of the Ideal Caribbean Person as articulated by the CARICOM Heads of Government in the following areas: “demonstrate multiple literacies, independent and critical thinking and innovative application of science and technology to problem solving. Such a person should also demonstrate a positive work attitude and value and display creative imagination and entrepreneurship”. In keeping with the UNESCO Pillars of Learning, on completion of this course of study, students will learn to do, learn to be and learn to transform oneself and society.

♦ AIMS

This syllabus aims to:

1. improve on the mathematical knowledge, skills and techniques with an emphasis on accuracy;

2. empower students with the knowledge, competencies and attitudes which are precursors for academia as well as quality leadership for sustainability in the dynamic world;

3. provide students with the proficiencies required to model practical situations and provide workable solutions in their respective fields of work and study;

4. develop competencies in critical and creative thinking, problem solving, logical reasoning, modelling, team work, decision making, research techniques and information communication and technology for life-long learning;

5. nurture desirable character qualities that include self-confidence, self-esteem, ethics and emotional security;

6. make Mathematics interesting, recognisable and relevant to the students locally, regionally and globally.

♦ SKILLS AND ABILITIES TO BE ASSESSED

The skills and abilities that students are expected to develop on completion of this syllabus have been group under three headings:

(a) Conceptual Knowledge;

(b) Algorithmic Knowledge; and,

(c) Reasoning.

Conceptual Knowledge

The examination will test candidates' ability to recall, select and use appropriate facts, concepts and principles in a variety of contexts.

Algorithmic Knowledge

The examination will test candidates' ability to manipulate mathematical expressions and procedures using appropriate symbols and language, logical deduction and inferences.

Reasoning

The examination will test candidates' ability to select appropriate strategy or select, use and evaluate mathematical models and interpret the results of a mathematical solution in terms of a given real-world problem and engage in problem-solving.

♦ PREREQUISITES OF THE SYLLABUS

Any person with a **good** grasp of the contents of the syllabus of the Caribbean Secondary Education Certificate (CSEC) General Proficiency course in Mathematics, or equivalent, should be able to undertake the course. However, successful participation in the course will also depend on the possession of good verbal and written communication skills.

STRUCTURE OF THE SYLLABUS

The Integrated Mathematics Syllabus comprises three Modules, each requiring at least 50 hours. Students will develop the skills and abilities identified through the study of:

Module 1 - FOUNDATIONS OF MATHEMATICS

Module 2 - STATISTICS

Module 3 - CALCULUS

♦ RESOURCES FOR ALL MODULES

Backhouse, J.K., and Houldsworth, S.P.T.	*Pure Mathematics Book 1: A First Course*. London: Longman Group Limited, 1981.
Bostock, L., and Chandler, S.	*Core Maths for Advanced Level 3rd Edition*. London: Stanley Thornes (Publishers) Limited, 2000.
Campbell, E.	*Pure Mathematics for CAPE: Volume 1*, Kingston: LMH Publishing Limited. 2007.
Dakin, A., and Porter, R.I.	*Elementary Analysis*. London: Collins Educational, 1991.
Hartzler, J.S., and Swetz, F.	*Mathematical Modelling in the Secondary School Curriculum: A Resource Guide of Classroom Exercises*. Vancouver: National Council of Teachers of Mathematics, Incorporated, Reston, 1991.
Martin, A., Brown, K., Rigsby, P., and Ridley, S.	*Advanced Level Mathematics Tutorials Pure Maths CD-ROM (Trade Edition), Multi-User Version and Single User version*. Cheltenham: Stanley Thornes (Publishers) Limited, 2000.
Stewart, J.	*Calculus 7th Edition*. Belmont: Cengage Learning, 2011
Talbert, J.F., and Heng, H.H.	*Additional Mathematics Pure and Applied 6th Edition*. Singapore: Pearson Educational. 2010.
Wolfram Mathematica (software)	

♦ MODULE 1: FOUNDATIONS OF MATHEMATICS

GENERAL OBJECTIVES

On completion of this Module, students should:

1. acquire competency in the application of algebraic techniques;
2. appreciate the role of exponential or logarithm functions in practical modelling situations;
3. understand the importance of relations functions and graphs in solving real-world problems;
4. appreciate the difference between a sequence and a series and their applications;
5. appreciate the need for accuracy in performing calculations;
6. understand the usefulness of different types of numbers.

SPECIFIC OBJECTIVES		CONTENT & SKILLS
1.	**Numbers**	
Students should be able to:		
1.1	distinguish among the sets of numbers;	Real and complex numbers; Identifying the set of complex number as the superset of other numbers; Real and imaginary parts of a complex number $z = x + iy$
1.2	solve problems involving the properties of complex numbers;	Equality, conjugate, modulus and argument. Addition, subtraction, multiplication and division (realising the denominator).
1.3	represent complex numbers using the Argand diagram;	Represent complex numbers, the sum and difference of two complex numbers.
1.4	find complex solutions, in conjugate pairs, to quadratic equations which has no real solutions.	Solving quadratic equations where the discriminant is negative.

SPECIFIC OBJECTIVES		CONTENT & SKILLS

2. Coordinate Geometry

Students should be able to:

2.1	solve problems involving concepts of coordinate geometry;	Application of: gradient; length and mid-point of a line segment; equation of a straight line.
2.2	relate the gradient of a straight line to the angle it makes with the horizontal line.	If $y = mx + c$, then $\tan\theta = m$, where θ is the angle made with the positive x-axis.

3. Functions, Graphs, Equations and Inequalities

Students should be able to:

3.1	combine components of linear and quadratic functions to sketch their graphs;	Intercepts, gradient, minimum/maximum point and roots.
3.2	determine the solutions of a pair of simultaneous equations where one is linear and the other is nonlinear;	Graphical and algebraic solutions. Equations of the form $axy + bx + cy = d$ and $ex^2 + fy = g$, where $a, b, c, d, e, f, g \in \mathbb{R}$
3.3	apply solution techniques of equations to solve real life problems;	Worded problems including quadratic equations, supply and demand functions and equations of motion in a straight line.
3.4	determine the solution set for linear and quadratic inequalities;	Graphical and algebraic solutions.
3.5	solve equations and inequalities involving absolute linear functions;	Equations and inequalities of type $\lvert ax + b\rvert \leq c \Rightarrow \dfrac{-c-b}{a} \leq x \leq \dfrac{c-b}{a}$, where $a, b, c \in\backslash\mathbb{R}$. Worded problems.
3.6	determine an invertible section of a function;	Functions that are invertible for restricted domains. Quadratic functions and graphs. Domain and range of functions and their inverse.
3.7	evaluate the composition of functions for a given value of x.	Addition, subtraction, multiplication and division of functions for example $h(x) = \frac{f(x)}{g(x)}$ Composite function: $h(x) = g[f(x)]$ Solving equations and finding function values.

MODULE 1: FOUNDATIONS OF MATHEMATICS (cont'd)

SPECIFIC OBJECTIVES		CONTENT & SKILLS
4.	**Logarithms and Exponents**	
Students should be able to:		
4.1	apply the laws of indices to solve exponential equations in one unknown;	Equations of type $a^{f(x)} = b^{g(x)}$, where $f\ and\ g$ are linear or quadratic .polynomials.
4.2	identify the properties of exponential and logarithmic functions;	Sketching the graphs of exponential (base e) and logarithmic functions (bases 10 and e).
4.3	simplify logarithmic expressions using the laws of logarithm;	Laws of logarithm excluding $log_a b = \frac{log_c b}{log_c a}$
4.4	identify the relationship between exponents and logarithms;	$y = log_a x \Leftrightarrow a^y = x$
4.5	convert between the exponential and logarithmic equations;	
4.6	apply the laws of logarithms to solve equations involving logarithmic expressions;	Equations of the type $a\,log(x+b) = log(c) - d$
4.7	solve problems involving exponents and logarithms.	Equations of type $a^x = b$ and $log_a x = b,$ where $a = 10$ or e Converting equations to linear form: $y = ax^b \Leftrightarrow log\,y = log\,a + b\,log\,x$ $y = ab^x \Leftrightarrow log\,y = log\,a + x\,log\,b$
5.	**Remainder and Factor Theorem**	
Students should be able to:		
5.1	state the remainder and factor theorem;	If $f(a) = 0$, then $(x-a)$ is a factor of f. If $f(b) \neq 0$, then $(x-b)$ leaves remainder $r \in \mathbb{R}$ when it divides f.
5.2	divide polynomials up to the third degree by linear expressions;	Methods of long division and inspection.

MODULE 1: FOUNDATIONS OF MATHEMATICS (cont'd)

SPECIFIC OBJECTIVES		CONTENT & SKILLS
Remainder and Factor Theorem (cont'd)		
Students should be able to:		
5.3	solve problems involving the factor and remainder theorems.	If $(x - a)$ is a factor of the polynomial $f(x)$, then $f(x)$ has a root at $x = a$ Including finding coefficients of a polynomial given a factor or remainder when divided by a linear expression. Factorising cubic polynomials where one factor can be found by inspection.
6.	**Sequences and Series**	
Students should be able to:		
6.1	solve problems involving the binomial expansion;	$(a + b)^n$ where n is a positive integer not greater than 3 and $a, b \in \mathbb{R}$. Problems involving finding the terms or the coefficient of a term of an expansion of linear expression such as $(ax + b)^3$. While the students may become familiar with Pascal triangle and notations relating to $\binom{n}{r}$, it is sufficient to know the binomial coefficients 1-2-1 and 1-3-3-1 for examination purposes.
6.2	identify arithmetic and geometric progressions;	Common ratio and common difference.
6.3	evaluate a term or the sum of a finite arithmetic or geometric series;	Problems including applications to simple and compound interest, annually to quarterly
6.4	determine the sum to infinity for geometric series;	$-1 < r < 1$

MODULE 1: FOUNDATIONS OF MATHEMATICS (cont'd)

SPECIFIC OBJECTIVES		CONTENT & SKILLS
7.	**Matrices and Systems of Equations**	
Students should be able to:		
7.1	perform the basic operations on matrices;	Addition, subtraction, multiplication, scalar multiple, equality of matrices.
7.2	represent data in matrix form;	System of equations, augmented matrix.
7.3	evaluate the determinant of a 3x3 matrix;	
7.4	solve a system of three linear equations with three unknowns using Cramer's rule;	While the examination questions will be structured to make the Cramer's rule convenient to use, solutions by alternative methods will be accepted.
8.	**Trigonometry**	
Students should be able to:		
8.1	evaluate sine, cosine and tangent of an angle given in radians;	Converting between degrees and radian measure of an angle.
8.2	solve equations involving trigonometric functions;	Functions of the form $a\sin(x+b)=c$ $a\cos(x+b)=c$ $a\tan(x+b)=c$ where the domain is a subset of $-2\pi \leq x \leq 2\pi$ Principal, secondary and other solutions obtained by using the general solution.
8.3	identify the graph of the sine, cosine and tangent functions;	Characteristics of graphs with standard periods.
8.4	solve problems involving the graphs of trigonometric functions.	Problems including locating intercepts and roots, minimum and maximum values for a domain

Suggested Teaching and Learning Activities

To facilitate students' attainment of the objectives in this Module, teachers are advised to engage students in the following teaching and learning activities.

1. Engage students in discussion on the meaning of the square root of a negative number.
2. Engage students in activities that facilitate the transfer knowledge of real numbers and vectors to operations with complex numbers.
3. Allow students to use triangles and squares to verify trigonometric ratios and Pythagoras theorem.

MODULE 1: FOUNDATIONS OF MATHEMATICS (cont'd)

4. Engage students in activities that will allow them to relate complex solutions of quadratic equations to the quadratic graph that has no x-intercept.

5. Allow student to apply geometrical triangle properties to determine the modulus and argument of a complex number.

6. Use teaching tools that include ICT and graphical software such as Wolfram Mathematica (wolfram.com) and geogebra (geogebra.com) software to draw graphs of functions and investigate their behaviour.

7. Give students activities that involve constructing lines of symmetry and use vertical and horizontal line tests to investigate invertible functions.

8. Use online generic calculators that include https://www.symbolab.com/solver/.

9. Use series calculators that include http://calculator.tutorvista.com/geometric-sequence-calculator.html or even http://www.wikihow.com/Find-the-Sum-of-a-Geometric-Sequence.

10. Use matrix calculators and support learning activities that involve matrices. Online matrix calculator sites include, http://www.bluebit.gr/matrix-calculator/, http://www.mathsisfun.com/algebra/matrix-calculator.html.

11. Use graphing utility to create models of graphs and shapes. Then adjust these models and observe how the parameters of the equations change.

RESOURCES

Caribbean Examinations Council	Injective and Surjective Functions: Barbados, 1998.
Caribbean Examinations Council	The Real Number System: Barbados, 1997

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MODULE 2: STATISTICS

GENERAL OBJECTIVES

On completion of this Module, students should:

1. understand the concept of randomness and its role in sampling and collection, description and analysis of data;
2. appreciate that the numerical and graphical representation of data is an important part of data analysis;
3. understand the concept of probability and its applications to real-world situations;
4. appreciate data-analysis processes for applications to real-world situations.

SPECIFIC OBJECTIVES	CONTENT & SKILLS
1. Data and Sampling	
Students should be able to:	
1.1 distinguish between sample and population;	Sample survey and Population census, Statistics and parameters: Statistics: mean ($\bar{x}$), variance(s^2) Parameter: mean μ and variance (σ^2) Sample relevant to population characteristics (sample error).
1.2 distinguish among sampling methods;	Random number generators, random number table; Simple random, stratified, systematic, and cluster.
1.3 select sampling method relevant to population characteristics;	Representative sample Sample error.
2. Presentation of Data	
Students should be able to:	
2.1 organise raw data into tabular form;	Tally tables, frequency tables, cumulative frequency tables.

MODULE 2: STATISTICS (cont'd)

SPECIFIC OBJECTIVES		CONTENT & SKILLS
Presentation of Data (cont'd)		
Students should be able to:		
2.2	present data in a variety of forms;	Bar chart, pie chart, line graph, histogram, cumulative frequency graph, stem-and-leaf, box-and-whiskers plot, scatter plots; Cross-tabulations of nominal/categorical data.
2.3	identify the shape of a distribution;	Symmetric, positively skewed and negatively skewed
3.	**Measures of Location and Spread**	
Students should be able to:		
3.1	select measures of location for appropriate data types;	Measures of central tendencies: mean, mode and median; discrete and continuous variable; suitability of measures of location to nominal, ordinal, interval and ratio scales of data; advantages and disadvantages of different measures of location.
3.2	determine measures of location for ungrouped data;	
3.3	determine estimates for measures of location for grouped data;	Including median and mode. See formula sheet.
3.4	Select measures of spread for appropriate data types.	Measures of Dispersion: range, standard deviation and Interquartile range (IQR) where quartile one and quartile two are the median of the lower and upper halves respectively. Advantages and disadvantages of different measures of spread. The empirical rule for the standard deviation in relation to the mean.
3.5	determine measures of spread for ungrouped data;	Ranges, variance, standard deviation.
3.6	determine estimates for measures of spread for grouped data;	

SPECIFIC OBJECTIVES		**CONTENT & SKILLS**
4.	**Permutations and Combinations**	
Students should be able to:		
4.1	calculate permutations of numbers;	Arrangement of n distinct items, or of r items from a total of n distinct items, nPr. Consider also cases where items are repeated.
4.2	calculate combinations of numbers;	Number of possible groupings of r items from a total of n distinct items, nCr.
4.3	use counting techniques to solve real-life problems;	
5.	**Probability, Probability Distributions and Regression**	
Students should be able to:		
5.1	distinguish among the terms experiment, outcome, event, sample space;	Concept of probability. Sum of probabilities, sample space and complementary events.
5.2	apply basic rules of probability;	Probability formulae: addition and multiplication. Types of events: mutually exclusive and mutually exhaustive, independent and dependent. Conditional probability: contingency tables, probability tree diagrams (with and without replacement).
5.3	explain the meaning of calculated probability values;	Theoretical vs. experimental probability. Percentages and relative frequencies.
5.4	investigate random variables;	Concept of a random variable. Discrete random variable; Distribution table of a random variable with maximum 5 possible values. Expectation, variance and standard deviation.

SPECIFIC OBJECTIVES | **CONTENT & SKILLS**

Probability, Probability Distributions and Regression (cont'd)

Students should be able to:

SPECIFIC OBJECTIVES		CONTENT & SKILLS
5.5	calculate probabilities from discrete probability distribution table;	Solving problems involving probabilities and expected values where, for example, the probability of an event is unknown.
5.6	solve problems involving the binomial distribution;	Binomial formula and binomial table of probabilities for at most ten trials.
5.7	determine characteristics of a Normal distribution;	Properties of the normal distribution curve; *z*-scores.
5.8	determine percentages of a population within desired limits of standard deviation;	Finding probabilities and *z*-values using the normal table.
5.9	investigate linear regression;	Concept of correlation; Regression line
5.10	evaluate correlation coefficient given summary statistics;	Substituting summary statistics in the formula for r. See formula sheet.
5.11	interpret the value of the correlation coefficient;	Negative, positive and no correlation. Strong, moderate and weak correlation.
5.12	draw an estimated regression line on a scatter plot;	Scatter plots, regression lines of best fit.
5.13	determine the equation of a regression line using summary statistics;	Substituting summary statistics for *a* and *b* in the equation $\hat{y}_i = a + bx_i$. See formula sheet.
5.14	Use statistics to solve real-world problems;	Case studies.
5.15	interpret results of statistical calculations.	

Suggested Teaching and Learning Activities

To facilitate students' attainment of the objectives in this Module, teachers are advised to engage students in the following teaching and learning activities.

1. Show videos from YouTube to reinforce concepts especially the more difficult topics – in this way, the students will hear a different voice, and see concepts from a different perspectives.

2. Encourage students to use computer applications to draw graphs and charts and perform calculations.

3. Use ICT devices such as projectors and telephones to make presentations and share data.

4. Show documentaries or movies on real-life application of statistics (for example, the movie Money Ball).

5. Use case studies taken from real-life events, for example, examination results, company reports and police reports, which can be obtained from the printed media or Internet to teach related concepts.

6. Take student on a field trip to see how Mathematics is used in the workplace such as to the central statistical office, workplace of actuaries, or research companies.

7. Invite a guest speaker, for example, a lecturer from a university, to talk to the students about Mathematics, its uses, abuses, misunderstandings, relevance, benefits, and careers.

8. Engage in web conferencing among teachers as well as students from across the region.

9. Exchange visits with a teacher and class from another school. Your class can visit the other school where you and another teacher can team teach a topic to both groups. At another time, invite the other class over to your school and again team teach.

10. Collect data out of the classroom or school by observing some event. Students will then apply mathematical operations on the data.

11. Allow students to research a topic and then present to the class.

12. Use worksheets to reinforce and practice different subtopics.

13. Assign class projects to design models of buildings or other items – drawn to scale, utilising formulae, and other mathematical concepts.

14. Use the acronym COPAI as a step-by-step approach to introduce students to descriptive statistics: collection, organisation, presentation, analysis, and interpretation of data.

15. Allow students to design questionnaires and develop questions for interviews.

16. Use online generic calculators that include https://www.symbolab.com/solver/

17. Become a member of a statistical association.

    For example:
    American Statistical Association (ASA) – http://www.amstat.org/index.cfm. The American Statistical Association publishes scholarly journals; statistical magazines; and a variety of conference proceedings, books, and other materials related to the practice of statistics.

    For example:
    Ethic in Statistics http://www.amstat.org/about/ethicalguidelines.cfm.
    Journal of Statistics Education http://www.amstat.org/publications/jse/ is a free, online, international journal focusing on the teaching and learning of statistics. This site also contains links to several statistical education organisations, newsletters, discussion groups and the JSE Dataset Archives.

    Useful sites for teachers:
    http://www.amstat.org/education/usefulsitesforteachers.cfm
    Royal Statistical Society http://www.rss.org.uk/

18. Participate **in competitions** 2015 Student Research Paper Contest

    The online journal PCD is looking for high-school, undergraduate, and graduate students, as well as medical residency and postdoctoral fellows to submit papers.

19. Secure an internship program http://stattrak.amstat.org/2014/12/01/2015-internships/
    STATtr@k is geared toward individuals who are in a statistics program, recently graduated from a statistics program, or recently entered the job world.

20. Teach yourself tutorials http://stattrek.com/

21. Subscribe to a statistics magazine e.g. Significance magazine http://www.statslife.org.uk/about-significance-mag articles are written by statisticians for anyone with an interest in the analysis and interpretation of data.

RESOURCES

http://www.worldofstatistics.org/
http://www.worldofstatistics.org/primary-secondary-school-teacher-resources/
http://www.examiner.com/article/community-college-students-and-the-international-year-of-statistics
https://www.youtube.com/watch?v=yxXsPc0bphQ&list=PLA6598DFE68727A9C
https://www.youtube.com/watch?v=HKA0htesJOA

♦ MODULE 3: CALCULUS

GENERAL OBJECTIVES

On completion of this Module, students should:

1. develop curiosity in the study of limits and continuity;
2. appreciate the importance of differentiation and integration in analysing functions and graphs;
3. enjoy using calculus as a tool in solving real-world problems.

SPECIFIC OBJECTIVES	CONTENT & SKILLS
1. Limits and Continuity	
Students should be able to:	
1.1 describe the limiting behaviour of a function of x, as x approaches a given number;	
1.2 use limit notation;	$\lim_{x \to a} f(x) = L, or$ $f(x) \to L \, as \, x \to a$
1.3 evaluate limits using simple limit theorems;	If $\lim_{x \to a} f(x) = F$, $\lim_{x \to a} g(x) = G$ and k is a constant, then $\lim_{x \to a} kf(x) = kF$, $\lim_{x \to a} f(x)g(x) = FG$, $\lim_{x \to a} \{f(x) + g(x)\} = F + G$, and, provided $G \neq 0$, $\lim_{x \to a} \frac{f(x)}{g(x)} = \frac{F}{G}$
1.4 apply factorisation to expressions whose limits are indeterminate;	Indeterminate forms. Polynomials which can be factorised.
1.5 apply the concept of left and right-handed limits to continuity;	Definition of continuity. Graphs of continuous functions.

MODULE 3: CALCULUS (cont'd)

SPECIFIC OBJECTIVES		CONTENT & SKILLS
Limits and Continuity (cont'd)		
Students should be able to:		
1.6	identify the points for which a function is discontinuous;	Discontinuous graphs. Piece-wise functions.
2.	**Differentiation**	
2.1	relate the derivative of a function with the gradient at a point on that function;	The gradient - derivative relationship.
2.2	use notations for the first derivative of a function, y = f(x);	y', $\mathrm{f}'(x)$ and $\frac{\mathrm{d}y}{\mathrm{d}x}$
2.3	differentiate polynomials;	Differentiation from first principle not required. The derivative of x^n. $\frac{\mathrm{d}}{\mathrm{d}x}(x^n) = n\,x^{n-1}$ where n is any real number $\frac{\mathrm{d}}{\mathrm{d}x} c\,\mathrm{f}(x) = c\frac{\mathrm{d}}{\mathrm{d}x}\,\mathrm{f}(x)$ where c is a constant $\frac{\mathrm{d}}{\mathrm{d}x}[f(x) \pm g(x)] = \frac{\mathrm{d}}{\mathrm{d}x}[f(x)] \pm \frac{\mathrm{d}}{\mathrm{d}x}[g(x)]$
2.4	differentiate expressions involving sine and cosine functions;	$\frac{\mathrm{d}}{\mathrm{d}x}(sin\,x) = cos\,x$ and $\frac{\mathrm{d}}{\mathrm{d}x}(cos\,x) = -\,sin\,x;$
2.5	apply the chain rule in the differentiation of composite functions;	Powers of a function; function of a function.
2.6	differentiate exponential and logarithmic functions;	The derivative of e^u and $ln\,u$ where u is a function of x
2.7	differentiate products and quotients;	Polynomials, sine, cosine, e^x and $ln\,x$.
2.8	determine the stationary point(s) of a given function;	Minimum and maximum points and point of inflexion.

MODULE 3: CALCULUS (cont'd)

SPECIFIC OBJECTIVES		CONTENT & SKILLS
Differentiation (cont'd)		
Students should be able to:		
2.9	obtain the second derivative of a function;	$If\ y = f(x), y'' = \frac{d^2y}{dx^2} = f''(x) = \frac{d}{dx}\left(\frac{dy}{dx}\right)$
2.10	investigate the nature of the stationary points;	Maximum and minimum points. $f''(x_0) > 0$ indicates minimum value. $f''(x_0) < 0$ indicates maximum value. $f''(x_0) = 0$ possibly a point of inflexion If $f''(x_0) = 0$ then test the sign of the gradient on either side of the stationary point to determine its nature..
3.	**Application of Differentiation**	
3.1	apply the concept of the derivative to rate of change;	$\frac{dy}{dx} \approx \frac{\Delta y}{\Delta x}$ Problems including: cost, revenue and profit functions.
3.2	solve problems involving rates of change;	Related rates.
3.3	use the sign of the derivative to investigate where a function is increasing or decreasing;	Increasing when $f'(x) > 0$ Decreasing when $f'(x) < 0$
3.4	solve problems involving stationary points ;	Point(s) of inflexion not included.
3.5	apply the concept of stationary (critical) points to curve sketching;	Polynomials of degree 3.
3.6	find the first partial derivative of a function of two variables;	Notations: $f_x, f_y, \frac{\partial}{\partial x}[f(x,y)], \frac{\partial}{\partial y}[f(x,y)]$ Interpret partial derivatives.
3.7	solve problems involving differentiation;	Applications to the agriculture, social sciences, physical sciences, engineering and other areas.

4. Integration

4.1	define integration as the reverse process of differentiation;	Derivative-integral relationship.
4.2	compute indefinite integrals of polynomials;	$\int x^n \, dx = \frac{x^{n+1}}{n+1} + C, n \neq -1, n \in \mathbb{R}$
4.3	integrate expressions that involve trigonometric functions;	$\int (a \sin x) dx$, $\int (a \cos x) dx$, $\int cos(ax+b)\, dx$
4.4	integrate functions of the form $\frac{1}{p(x)}$, where $p(x)$ is a linear polynomial;	$\int \frac{\mathbf{1}}{\boldsymbol{ax+b}} \boldsymbol{dx} = \frac{\boldsymbol{ln\|ax+b\|}}{\boldsymbol{a}} + \boldsymbol{c}$
4.5	integrate composite functions by substitution;	$\int f[g(x)]\, dx$, where $g(x)$ is linear.
4.6	compute definite integrals;	$\int_a^b (ax^3 + bx^2 + cx + d)\, dx$ $\int_a^b f(x)\, dx = F(b) - F(a)$, where $F'(x) = f(x)$
4.7	apply integration to determine the area between a curve and a straight line;	$\int_a^b [f(x) - g(x)]\, dx$; one example is the area between a curve and x-axis
4.8	solve first order differential equations;	Restricted to variables separable.
4.9	solve problems involving integration;	Application to a variety of academic disciplines.

<u>Suggested Teaching and Learning Activities</u>

To facilitate students' attainment of the objectives in this Module, teachers are advised to engage students in the following teaching and learning activities.

1. Use online generic calculators that include https://www.symbolab.com/solver/

2. Use graphing utility to create models of graphs and shapes. Then adjust these models and observe how the equations change.

3. Organise debates on situations that involve mathematics, for example, the utility of mathematics to other disciplines.

4. Browse fun calculus activities online, for example, at http://teachinghighschoolmath.blogspot.com/2013/03/fun-calculus-ap-activities.html.

MODULE 3: CALCULUS (cont’d)

5. Use online teaching resources such as videos and PowerPoint (ppt) presentations from youtube and google search.

RESOURCES

http://teachinghighschoolmath.blogspot.com/2013/03/fun-calculus-ap-activities.html

♦ OUTLINE OF ASSESSMENT

The same scheme of assessment will be applied to each Module of this single-unit course. Candidates' performance will be reported as an overall grade and a grade on each Module.

The assessment will comprise two components:

1. External assessment undertaken at the end of the academic year in which the course is taken. This contributes 80% to the candidate's overall grade.

2. School-Based assessment undertaken throughout this course. This contributes 20% to the candidate's overall grade.

EXTERNAL ASSESSMENT (80%)

The candidate is required to sit a multiple choice paper and a written paper for a total of 4 hours.

Paper 01 (1 hour 30 minutes)	This paper comprises 45 compulsory multiple choice items, 15 from each module. Each item is worth 1 mark.	**30%**
Paper 02 (2 hours 30 minutes)	This paper consists of three sections, each corresponding to a Module. Each section will contain two extended-response questions. Candidates will be required to answer all six questions.	**50%**

SCHOOL-BASED ASSESSMENT (SBA) (20%)

Paper 031

The School-Based Assessment comprises a project designed and internally assessed by the teacher and externally moderated by CXC. This paper comprises a single project requiring candidates to demonstrate the practical application of Mathematics in everyday life. In essence it should allow candidates to probe, describe and explain a mathematical area of interest and communicate the findings using mathematical symbols, language and tools. The topic(s) chosen may be from any Module or combination of different Modules of the syllabus. The project may require candidates to collect data (Project B), or may be theory based (Project A), requiring solution or proof of a chosen problem.

Project A is based on applying mathematical concepts and procedures from any module in the syllabus in order to understand, describe or explain a real world phenomenon. The project is theory based. Project A is based on applying mathematical concepts and procedures from any module in the syllabus in order to understand, describe or explain a real world phenomenon. The project is experiment based and involves the collection of data.

Candidates should complete one project, either Project A or Project B.

Paper 032 (Alternative to Paper 031), examined externally

Paper 032 is a written paper consisting of a case study based on the three modules.
This paper is an alternative for Paper 031 and is intended for private candidates. Details are on page 27.

MODERATION OF THE SCHOOL-BASED ASSESSMENT

School-Based Assessment Record Sheets are available online via the CXC's website www.cxc.org.

All School-Based Assessment Record of marks must be submitted online using the SBA data capture module of the Online Registration System (ORS). A sample of assignments will be requested by CXC for moderation purposes. These assignments will be re-assessed by CXC Examiners who moderate the School-Based Assessment. Teachers' marks may be adjusted as a result of moderation. The Examiners' comments will be sent to schools. All samples must be delivered by the stipulated deadlines.

Copies of the students' assignments that are not submitted must be retained by the school until three months after publication by CXC of the examination results.

ASSESSMENT DETAILS

External Assessment by Written Papers (80% of Total Assessment)

Paper 01 (1 hour 30 minutes - 30% of Total Assessment)

1. **Composition of papers**

   (a) This paper consists of 45 multiple choice items and is partitioned into three sections (Module 1, 2 and 3). Each section contains 15 questions.

   (b) All items are compulsory.

2. **Syllabus Coverage**

   (a) Knowledge of the entire syllabus is required.

   (b) The paper is designed to test candidates' knowledge across the breadth of the syllabus.

3. **Question Type**

   Questions may be presented using words, symbols, tables, diagrams or a combination of these.

4. **Mark Allocation**

   (a) Each item is allocated 1 mark.

   (b) Each Module is allocated 15 marks.

   (c) The total number of marks available for this paper is 45.

   (d) This paper contributes 30% towards the final assessment.

5. **Award of Marks**

   Marks will be awarded for reasoning, algorithmic knowledge and conceptual knowledge.

   Reasoning: Clear reasoning, explanation and/or logical argument.

Algorithmic knowledge: Evidence of knowledge, ability to apply concepts and skills, and to analyse a problem in a logical manner.

Conceptual knowledge: Recall or selection of facts or principles; computational skill, numerical accuracy and acceptable tolerance in drawing diagrams.

6. **Use of Calculators**

    (a) Each candidate is required to have a silent non-programmable calculator and is responsible for its functioning.

    (b) The use of calculators with graphical displays will not be permitted.

    (c) Calculators must not be shared during the examination.

Paper 02 (2 hours 30 minutes - 50% of Total Assessment)

This paper will be divided into three sections, each section corresponding to a Module.

1. **Composition of Paper**

    (a) This paper consists of *six* questions, two questions from each Module.

    (b) All questions are compulsory.

2. **Syllabus Coverage**

    (a) Each question may require knowledge from more than one topic in the Module from which the question is taken and will require sustained reasoning.

    (b) Each question may address a single theme or unconnected themes.

    (c) The intention of this paper is to test candidates' in-depth knowledge of the syllabus.

3. **Question Type**

    Paper 02 consists of six essay type questions which require candidates to provide an extended response involving higher order thinking skills such as application, analysis, synthesis and evaluation.

    (a) Questions may require an extended response.

    (b) Questions may be presented using words, symbols, diagrams, tables or combinations of these.

4. **Mark Allocation**

    (a) Each question is worth *25* marks.

(b) The number of marks allocated to each sub-question will appear in brackets on the examination paper.

(c) Each Module is allocated 50 marks.

(d) The total marks available for this paper is 150.

(e) The paper contributes 50% towards the final assessment.

5. **Award of Marks**

(a) Marks will be awarded for reasoning, algorithmic knowledge and conceptual knowledge.

<u>Reasoning:</u>	Clear reasoning, explanation and/or logical argument.
<u>Algorithmic knowledge</u>:	Evidence of knowledge, ability to apply concepts and skills, and to analyse a problem in a logical manner.
<u>Conceptual knowledge</u>:	Recall or selection of facts or principles; computational skill, numerical accuracy and acceptable tolerance in drawing diagrams.

(b) Full marks are awarded for **correct** answers and the presence of **appropriate working**.

(c) It may be possible to earn partial credit for a correct method where the answer is incorrect.

(d) If an incorrect answer in an earlier question or part-question is used later in a section or a question, then marks may be awarded in the later part even though the original answer is incorrect. In this way, a candidate is not penalised twice for the same mistake.

(e) A correct answer given with no indication of the method used (in the form of written work) may receive (one) mark only. Candidates are, therefore, advised to show <u>all</u> relevant working.

6. **Use of Calculators**

(a) Each candidate is required to have a silent non-programmable calculator and is responsible for its functioning.

(b) The use of calculators with graphical displays will not be permitted.

(c) Answers found by using a calculator, without relevant working shown, may not be awarded full marks.

(d) Calculators must not be shared during the examination.

7. Use of Mathematical Tables

A booklet of mathematical formulae and tables will be provided.

SCHOOL-BASED ASSESSMENT (20 per cent)

School-Based Assessment is an integral part of the student assessment in the course of study covered by this syllabus. It is intended to assist the students in acquiring certain knowledge, skills and attitudes that are associated with the subject. The activities for the School-Based Assessment are linked to the syllabus and should form part of the learning activities to enable the student to achieve the objectives of the syllabus.

During the course of study for the subject, students obtain marks for the competence they develop and demonstrate in undertaking their School-Based Assessment assignments. These marks contribute to the final marks and grades that are awarded to the students for their performance in the examination.

The guidelines provided in this syllabus for selecting appropriate tasks are intended to assist teachers and students in selecting assignments that are valid for the purpose of School-Based Assessment. The guidelines provided for the assessment of these assignments are also intended to assist teachers in awarding marks that are reliable estimates of the achievements of students in the School-Based Assessment component of the course. In order to ensure that the scores awarded are in line with the CXC standards, the Council undertakes the moderation of a sample of the School-Based Assessment assignments marked by each teacher.

School-Based Assessment provides an opportunity to individualise a part of the curriculum to meet the needs of the student. It facilitates feedback to the student at various stages of the experience. This helps to build the self-confidence of the students as they proceed with their studies. School-Based Assessment also facilitates the development of the critical skills and abilities emphasised by this CAPE subject and enhances the validity of the examination on which the students' performance is reported. School-Based Assessment, therefore, makes a significant and unique contribution to both the development of the relevant skills and the testing and rewarding of the student for the development of those skills. Note that group work should be encouraged and employed where appropriate; however, candidates are expected to submit individual assignments for the School-Based Assessment.

REQUIREMENTS OF THE SCHOOL-BASED ASSESSMENT

The School-Based Assessment is based on skills and competencies related specifically to the Modules of that Unit. However, students who repeat in a subsequent sitting may reuse their School-Based Assessment marks.

Skills to be assessed

Reasoning:	Clear reasoning, explanation and/ or logical argument.
Algorithmic knowledge:	Evidence of knowledge, ability to apply concepts and skills, and to analyse a problem in a logical manner.
Conceptual knowledge:	Recall or selection of facts or principles; computational skill, numerical accuracy and acceptable tolerance in drawing diagrams.

Managing the research project

The research project is worth 20% of the candidate's total mark. Teachers should ensure that sufficient time is allowed for teaching the research skills required, explaining the requirements of the School-Based Assessment, discussing the assessment criteria and monitoring and evaluating the project work.

Planning

An early start to planning project work is highly recommended. A schedule of the dates for submitting project work (agreed by both teachers and candidates) should be established.

Length of the report

The length of the report should not exceed 1500 words, not including bibliography, appropriate quotations, sources, charts, graphs, tables, pictures, references and appendices.

CRITERIA FOR THE SCHOOL-BASED ASSESSMENT (Paper 031)

This paper is compulsory and consists of a project.

AIMS OF THE PROJECT

The aims of the project are to:

1. promote self-learning;

2. allow teachers the opportunity to engage in the formative assessment of their students;

3. enable candidates to use the methods and procedures of acquired to describe or explain real-life phenomena.

4. foster the development of critical thinking skills among students;

Requirements of the Project

The project will be presented in the form of a report and should include the following:

1. Project title
2. A statement of the problem
3. Identification of important elements of the problem
4. Mathematical Formulation of the problem or Research Methodology
5. Analysis and manipulation of the data
6. Discussion of findings

(a) **Integration of Project into the Course**

(i) The activities related to project work should be integrated into the course so as to enable candidates to learn and practice the skills of undertaking a successful project.

(ii) Some time in class should be allocated for general discussion of project work. For example, discussion of how data should be collected, how data should be analysed and how data should be presented.

(iii) Class time should also be allocated for discussion between teacher and student, and student and student.

(b) **Management of Project**

Planning

An early start to planning project work is highly recommended and the schedule of the dates for submission should be developed by teachers and candidates.

Length

The length of the report of the project should not exceed 1500 words (excluding diagrams, graphs, tables and references). A total of 10 percent of the candidate's score will be deducted for any research paper in excess of 1500 words (excluding diagrams, graphs, tables and references). If a deduction is to be made from a candidate's score, the teacher should clearly indicate on the assignment the candidate's original score before the deduction is made, the marks which are to be deducted, and the final score that the candidate receives after the deduction has been made.

Guidance

Each candidate should know the requirements of the project and its assessment process.

Although candidates may consult with resource persons besides the teacher the candidate's submission should be his or her own work.

Candidates are not expected to work on their own. The teacher is expected to give appropriate guidance at all stages of project work, for example, chapters to read, alternative procedures to follow and other sources of information.

Authenticity

Teachers are required to ensure that all projects are the candidates' work.

The recommended procedures are to:

1. engage candidates in discussion;
2. ask candidates to describe procedures used and summarise findings either orally or written;
3. ask candidates to explain specific aspects of the analysis.

ASSESSMENT CRITERIA FOR THE PROJECT

General

It is recommended that candidates be provided with assessment criteria before commencing the project.

1. The following aspects of the project will be assessed:

    (a) project title;

    (b) Introduction (purpose of project etc.);

    (c) Mathematical formulation;

    (d) Problem formulation;

    (e) Discussion of findings;

    (f) Overall presentation;

    (g) Reference/ bibliography;

    (h) List of references.

2. For each component, the aim is to find the level of achievement reached by the candidate.

3. For each component, only whole numbers should be awarded.

4. It is recommended that the assessment criteria be available to candidates at all times.

ASSESSING THE PROJECT

The project will be graded out of a total of 60 marks and marks will be allocated to each task as outlined below. Candidates will be awarded 2 marks for communicating information in a logical way using correct grammar. These marks are awarded under Task 7 below.

Allocation of Marks for the Research Project

Marks will be allocated according to the following scheme:

MARK SCHEME FOR THE SCHOOL-BASED ASSESSMENT

Project A

Project Descriptors	**Allocation of marks**	
Project Title [2]		
• Title is a clear statement and concise statement	2	(2)
• Title is a concise statement but not clear	1	
Introduction [5]		
• Problem is clearly stated	(1)	
• Purpose of Project		
○ Purpose is clearly stated	(1)	
○ Purpose relates to real-life situations **OR** Purpose relates to solving existing problem	(1)	
• Outcome can be readily implemented • Outcome can be implemented but with limitations	2 1	(2)
Mathematical Formulation [12]		
• Identifies all of the important elements of the problem and shows a complete understanding of the relationships among them. • Identifies some important elements of the problem and shows a general understanding of the relationships among them. • identifies some of the important elements of the problem and shows a very limited understanding of the relationships among them.	5 - 6 3 – 4 1 - 2	(6)
• Good understanding of the problem's mathematical concepts/principles • Fair understanding of the problem's mathematical concepts/principles • Limited understanding of the problem's mathematical concepts/principles	3 2 1	(3)
• Mathematical model(s)/method(s) applied is/are most suitable for the task • Mathematical model(s)/method(s) applied is/are appropriate for the task • Mathematical model(s)/method(s) applied is/are acceptable	3 2 1	(3)

Problem Solution [25]		
• Assumptions are clearly identified and explained. • Assumption is misidentified or stated in an unclear manner	2 1	(2)
• Logical algorithms are used and executed correctly • Algorithms are used but contain errors	3 – 4 1 - 2	(4)
• All diagrams are appropriate to the problem • Some of the diagrams are appropriate to the problem	2 1	(2)
• All diagrams are clearly labelled. • Some diagrams are clearly labelled	2 1	(2)
• Explanations are sufficient and clearly stated • Some explanations are sufficient and clearly stated • Most of the explanations are vague	4 2 – 3 1	(4)
• All of the theorems and/or formulae are relevant to the solution and correctly applied • Some of the theorems and/or formulae are relevant to the solution and correctly applied • Few of the theorems and/or formulae are relevant to the solution and not correctly applied	3 2 1	(3)
• More than 75% of calculations are accurate. • Between 50% and 75% of calculations are accurate	2 1	(2)
• Adequate reference to previous work • Limited reference to previous work given	2 1	(2)
• Interpretations of results are reasonable given the objectives, desired targets and research methodology. • Interpretation attempted but the interpretation does not refer back to the objectives or desired targets. The interpretations are not clearly supported by the methodology and/or results.	3- 4 1 - 2	(4)
Discussions of findings [10]		
• Discusses the validity of the solution	(1)	
• Applies the solution or proof correctly to the given real-world problem	(1)	
• Discussion is coherent, concise and relates to the purpose of the project • Discussion is coherent, concise but does not fully to the purpose of the project	2 1	(2)

• The limitations of the research are relevant and comprehensively discussed. • The limitations of the research are relevant but not fully discussed	2 1	(2)
• Recommendations are relevant and practical • Recommendations are relevant or practical	2 1	(2)
• Conclusion is succinct, fully reflects the objectives and is supported by data. • Conclusion is adequate, partially reflects the objectives and partially supported by data.	2 1	(2)
Overall Presentation [4]		
○ Communicates information in a logical way using correct grammar and appropriate mathematical jargon all of the time ○ Communicates information in a logical way using correct grammar and appropriate mathematical jargon most of the time ○ Communicates information in a logical way using correct grammar and appropriate mathematical jargon some of the time. ○ Communicates information in a logical way using correct grammar and appropriate mathematical jargon in a limited way.	4 3 2 1	(4)
Reference/Bibliography [2]		
In-text citing of previous work with references Inclusion of bibliography only	**2** **1**	**(2)**
TOTAL		**60 marks**

Project B

Project Descriptors (Project B)	Allocation of marks	
Project Title [2]		
Title is a clear statement	(1)	
Title is a concise statement	(1)	
Introduction [6]		
• Rationale for the project is logical • Rationale for the project is somewhat logical	2 1	(2)
• Problem(s)/ Objective(s) are clearly stated • Problem(s)/ Objective(s) not clearly stated	2 1	(2)
• Summary of research methodology adopted is succinctly stated (quantitative or qualitative) • Summary of research methodology adopted is somewhat incoherent (quantitative or qualitative)	2 1	(2)
Research Methodology [19]		
• Research method/design (experimental, quasi-experimental, non-experimental) is clearly and logically outlined. • Research method/design (experimental, quasi-experimental, and non-experimental) is not clearly documented.	2 1	(2)
• The limitations of the research are relevant and comprehensively discussed • The limitations of the research are relevant but not fully discussed	2 1	(2)
• Description of the sampling process/sample design (1) Identification of the target population (2) Specification of the sampling frame or otherwise justify (3) Description of sample selection methodology/ selection of subjects (participants) • sampling method (probability/random vs non-probability /non-random sampling) sample size is appropriate	 (1) (1) (1) (1)	
• Instrument Design Selection of instrument (e.g. questionnaires, interviews, case studies, tests, measures, observations, scales) is justified in a comparative manner. Selection of instrument (e.g. questionnaires, interviews, case studies, tests, measures, observations, scales) is justified but not comparative	 2 1	(2)

- Instrument has relevant items which are clearly articulated and are logically outlined (Alternatively, if a previously designed instrument is used then it must be cited and justified) Instrument has relevant items some which are clearly articulated (Alternatively, if a previously designed instrument is used then it must be cited and justified)	2 - 3 1	(3)
- Data Management (1) Data collection process is adequately described. (2) Data coding techniques (e.g. transferring item responses into numbers) are appropriate and clearly explained. (3) Data Entry/ Data Recording methods clearly described (4) Data Security method clearly described (Include information on the preservation of the database for this study e.g. backup measures)	(1) (1) (1) (1)	
Organization of Data (e.g. frequency tables) Concise discussion on the data extraction procedures from raw database into tabular form and inclusion of all tables in the report. Adequate discussion on the data extraction procedures from raw database into tabular form and the inclusion of some tables in the report.	2 1	(2)
Presentation of Findings[12]		
- Display of Results (e.g. Bar Graph, Pie Chart, Stem & Leaf Plot, Box and Whiskers Plot) ○ A variety of tables, graphs and figures are appropriately used according to the data type and portray the data accurately and clearly ○ A variety of tables, graphs and figures are appropriately used according to the data type and portray the data fairly accurately and clearly ○ A few tables, graphs and figures are used which portray the data with limited accuracy and clarity	5 - 6 3 - 4 1 - 2	(6)

• Description of tables, charts and figures: ○ Excellent description of the tables, graphs and figures. ○ Satisfactory description of the tables, graphs and figures. ○ Limited description of the tables, graphs and figures.	5 - 6 4 - 5 1 - 2	(6)
Analysis of Findings [15]		
Statistical analysis tools 1. Measures of Central Tendency 2. Measures of Variability 3. Measures of Relationship (e.g. correlation, regression) 4. Measures of Relative Position(e.g. percentiles, z-scores and t-scores) 5. Measure of dependence • An accurate discussion which includes calculations and meaningful comparisons of the findings using at least 3 of the statistical techniques. • A satisfactory discussion which includes calculations and meaningful comparisons of the findings using two of the statistical techniques. • A discussion which includes calculations and meaningful comparisons of the findings using one statistical technique.	3 2 1	(3)
• More than 75% of calculations are accurate. • Between 50% and 75% of calculations are accurate.	2 1	(2)
• Interpretations of results ○ An excellent interpretation of the results obtained, why they were obtained and identification of trends, patterns and anomalies. ○ An adequate or limited interpretation of the results, why they were obtained and identification of trends, patterns and anomalies.	3 - 4 1 - 2	(4)
• Recommendations for future development ○ Recommendations are relevant **and** practical ○ Recommendations are relevant **or** practical	2 1	(2)
○ Conclusion is comprehensive, reflects the hypothesis/objectives and is supported by data. ○ Conclusion is adequate, reflects the hypothesis/objectives and supported by data. Conclusion is satisfactory and reflects the hypothesis/objectives **OR** supported by data.	3 – 4 2 1	(4)

Overall Presentation [4]		
Communication of information in a logical way		
○ Communicates information in a logical way using correct grammar and appropriate mathematical jargon all of the time ○ Communicates information in a logical way using correct grammar and appropriate mathematical jargon most of the time ○ Communicates information in a logical way using correct grammar and appropriate mathematical jargon some of the time. ○ Communicates information in a logical way using correct grammar and appropriate mathematical jargons in a limited way.	4 3 2 1	(4)
Reference/Bibliography [2] In-text citing of previous work with references Inclusion of bibliography only	2 1	(2)
Appendix		

♦ REGULATIONS FOR PRIVATE CANDIDATES

Private candidates will be required to write Papers 01, 02 and 032. Detailed information on Papers 01 and 02 is given on pages 23–26 of this syllabus.

Paper 032 is the alternative paper to the School-Based Assessment. This paper is worth 20 per cent of the total mark for the Unit. Paper 032 will test the student's acquisition of the skills in the same areas of the syllabus identified for the School-Based Assessment. Consequently, candidates are advised to undertake a project similar to the project that the school candidates would normally complete and submit for School-Based Assessment to develop the requisite competences for this course of study. It should be noted that private candidates would not be required to submit a project document.

<u>Paper 032 (1 hour 30 minutes – 20 % of Total Assessment)</u>

1. **Composition of Paper**

   Paper 032 is a written paper consisting of a case study based on the three modules. The paper consists of three compulsory questions which are divided into parts. The questions test skills similar to those in the School-Based assessment (Paper 031).

2. **Syllabus Coverage**

   This paper is intended to test the knowledge and skills contained in Modules 1, 2 and 3 as outlined in the syllabus.

3. **Question Type**

   Questions in this paper may be short answer or essay type, based on the case study.

4. **Mark Allocation**

(i) This paper is worth 60 marks.
(ii) Each question is worth 20 marks and contributes 20 percent toward the final assessment.

♦ REGULATIONS FOR RESIT CANDIDATES

Resit candidates must complete Paper 01 and 02 of the examination for the year for which they re-register. A candidate who rewrites the examination within two years may reuse the moderated School-Based Assessment score earned in the previous sitting within the preceding two years.

Candidates are not required to earn a moderated score that is at least 50 per cent of the maximum possible score; any moderated score may be reused.

Candidates reusing SBA scores in this way must register as 'Resit candidates' and provide the previous candidate number. (In order to assist candidates in making decisions about whether or not to reuse a moderated SBA score, the Council will continue to indicate on the pre-slip if a candidate's moderated SBA score is less than 50%).

Resit candidates must be registered through a school, a recognised educational institution, or the Local Registrar's Office.

♦ ASSESSMENT GRID

The Assessment Grid for this Course contains marks assigned to papers and to Modules, and percentage contributions of each paper to total scores.

Papers	**Module 1**	**Module 2**	**Module 3**	**Total**	**(%)**
External Assessment **Paper 01** (1 hour 30 minutes) Multiple Choice	15 (30 weighted)	15 (30 weighted)	15 (30 weighted)	45 (90 weighted)	(30)
Paper 02 (2 hours 30 minutes) Extended Response	50	50	50	150	(50)
School-Based Assessment Paper 031 or Paper 032 (1 hour 30 minutes)	20	20	20	60	(20)
Total	100	100	100	300	(100)

♦ GLOSSARY OF EXAMINATION TERMS

WORD	DEFINITION	NOTES
Analyse	examine in detail	
Annotate	add a brief note to a label	Simple phrase or a few words only.
Apply	use knowledge/principles to solve problems	Make inferences/conclusions.
Assess	present reasons for the importance of particular structures, relationships or processes	Compare the advantages and disadvantages or the merits and demerits of a particular structure, relationship or process.
Calculate	arrive at the solution to a numerical problem	Steps should be shown; units must be included.
Classify	divide into groups according to observable characteristics	
Comment	state opinion or view with supporting reasons	
Compare	state similarities and differences	An explanation of the significance of each similarity and difference stated may be required for comparisons which are other than structural.
Construct	use a specific format to make and/or draw a graph, histogram, pie chart or other representation using data or material provided or drawn from practical investigations, build (for example, a model), draw scale diagram	Such representations should normally bear a title, appropriate headings and legend.
Deduce	make a logical connection between two or more pieces of information; use data to arrive at a conclusion	
Define	state concisely the meaning of a word or term	This should include the defining equation/formula where relevant.
Demonstrate	show; direct attention to...	

WORD	DEFINITION	NOTES
Derive	to deduce, determine or extract from data by a set of logical steps some relationship, formula or result	This relationship may be general or specific.
Describe	provide detailed factual information of the appearance or arrangement of a specific structure or a sequence of a specific process	Description may be in words, drawings or diagrams or any appropriate combination. Drawings or diagrams should be annotated to show appropriate detail where necessary.
Determine	find the value of a physical quantity	
Design	plan and present with appropriate practical detail	Where hypotheses are stated or when tests are to be conducted, possible outcomes should be clearly stated and/or the way in which data will be analysed and presented.
Develop	expand or elaborate an idea or argument with supporting reasons	
Diagram	simplified representation showing the relationship between components	
Differentiate/Distinguish (between/among)	state or explain briefly those differences between or among items which can be used to define the items or place them into separate categories	
Discuss	present reasoned argument; consider points both for and against; explain the relative merits of a case	
Draw	make a line representation from specimens or apparatus which shows an accurate relation between the parts	In the case of drawings from specimens, the magnification must always be stated.
Estimate	make an approximate quantitative judgement	
Evaluate	weigh evidence and make judgements based on given criteria	The use of logical supporting reasons for a particular point of view is more important than the view held; usually both sides of an argument should be considered.
Explain	give reasons based on recall; account for	

WORD	DEFINITION	NOTES
Find	locate a feature or obtain as from a graph	
Formulate	devise a hypothesis	
Identify	name or point out specific components or features	
Illustrate	show clearly by using appropriate examples or diagrams, sketches	
Interpret	explain the meaning of	
Investigate	use simple systematic procedures to observe, record data and draw logical conclusions	
Justify	explain the correctness of	
Label	add names to identify structures or parts indicated by pointers	
List	itemise without detail	
Measure	take accurate quantitative readings using appropriate instruments	
Name	give only the name of	No additional information is required.
Note	write down observations	
Observe	pay attention to details which characterise a specimen, reaction or change taking place; to examine and note scientifically	Observations may involve all the senses and/or extensions of them but would normally exclude the sense of taste.
Outline	give basic steps only	
Plan	prepare to conduct an investigation	
Predict	use information provided to arrive at a likely conclusion or suggest a possible outcome	
Record	write an accurate description of the full range of observations made during a given procedure	This includes the values for any variable being investigated; where appropriate, recorded data may be depicted in graphs, histograms or tables.

WORD	DEFINITION	NOTES
Relate	show connections between; explain how one set of facts or data depend on others or are determined by them	
Sketch	make a simple freehand diagram showing relevant proportions and any important details	
State	provide factual information in concise terms outlining explanations	
Suggest	offer an explanation deduced from information provided or previous knowledge. (... a hypothesis; provide a generalisation which offers a likely explanation for a set of data or observations.)	No correct or incorrect solution is presumed but suggestions must be acceptable within the limits of scientific knowledge.
Use	apply knowledge/principles to solve problems	Make inferences/conclusions.

♦ GLOSSARY OF MATHEMATICAL TERMS

WORDS	MEANING
Absolute Value	The absolute value of a real number x, denoted by $\|x\|$, is defined by $\|x\| = x$ if $x > 0$ and $\|x\| = -x$ if $x < 0$. For example, $\|-4\| = 4$.
Algorithm	A process consisting of a specific sequence of operations to solve a certain types of problems. See **Heuristic**.
Argand Diagram	An Argand diagram is a rectangular coordinate system where the complex number $x + iy$ is represented by the point whose coordinates are x and y. The x-axis is called the real axis and the y-axis is called the imaginary axis.
Argument of a Complex Number	The angle, $\theta = tan^{-1}\left(\frac{y}{x}\right)$, is called the argument of a complex number $z = x + iy$.
Arithmetic Mean	The average of a set of values found by dividing the sum of the values by the amount of values.
Arithmetic Progression	An arithmetic progression is a sequence of elements, $a1, a2, a3$,….., such that there is a common difference of successive terms. For example, the sequence {2, 5, 8, 11, 14, …} has common difference, $d = 3$.
Asymptotes	A straight line is said to be an asymptote of a curve if the curve has the property of becoming and staying arbitrarily close to the line as the distance from the origin increases to infinity.
Augmented Matrix	If a system of linear equations is written in matrix form $Ax = b$, then the matrix $[A\|b]$ is called the augmented matrix.
Average	The average of a set of values is the number which represents the usual or typical value in that set. Average is synonymous with measures of central tendency. These include the mean, mode and median.
Axis of symmetry	A line that passes through a figure such that the portion of the figure on one side of the line is the mirror image of the portion on the other side of the line.
Bar Chart	A bar chart is a diagram which is used to represent the frequency of each category of a set of data in such a way that the height of each bar if proportionate to the frequency of the category it represents. Equal space should be left between consecutive bars to indicate it is not a histogram

WORDS	MEANING
Base	In the equation $y = log_a x$, the quantity a is called the base. The base of a polygon is one of its sides; for example, a side of a triangle. The base of a solid is one of its faces; for example, the flat face of a cylinder. The base of a number system is the number of digits it contains; for example, the base of the binary system is two.
Bias	Bias is systematically misestimating the characteristics of a population (parameters) with the corresponding characteristics of the sample (statistics).
Biased Sample	A biased sample is a sample produced by methods which ensures that the statistics is systematically different from the corresponding parameters.
Bijective	A function is bijective if it is both injective and surjective; that is, both one-to-one and unto.
Bimodal	Bimodal refers to a set of data with two equally common modes.
Binomial	An algebraic expression consisting of the sum or difference of two terms. For example, $(ax + b)$ is a binomial.
Binomial Coefficients	The coefficients of the expansion $(x + y)^n$ are called binomial coefficients. For example, the coefficients of $(x + y)^3$ are 1, 3, 3 and 1.
Box-and-whiskers Plot	A box-and-whiskers plot is a diagram which displays the distribution of a set of data using the five number summary. Lines perpendicular to the axis are used to represent the five number summary. Single lines parallel to the axis are used to connect the lowest and highest values to the first and third quartiles respectively and double lines parallel to the axis form a box with the inner three values.
Categorical Variable	A categorical variable is a variable measured in terms possession of quality and not in terms of quantity.
Class Intervals	Non-overlapping intervals, which together contain every piece of data in a survey.
Closed Interval	A closed interval is an interval that contains its end points; it is denoted with square brackets $[a, b]$. For example, the interval $[-1,2]$ contains -1 and 2. For contrast see **open interval**.

WORDS	MEANING
Composite Function	A function consisting of two or more functions such that the output of one function is the input of the other function. For example, in the composite function $f(g(x))$ the input of f is g.
Compound Interest	A system of calculating interest on the sum of the initial amount invested together with the interest previously awarded; if A is the initial sum invested in an account and r is the rate of interest per period invested, then the total after n periods is $A(1+r)^n$.
Combinations	The term combinations refers to the number of possible ways of selecting r objects chosen from a total sample of size n if you don't care about the order in which the objects are arranged. Combinations is calculated using the formula $nCr = \binom{n}{r} = C_r^n = \frac{n!}{r!(n-r)!}$. See **factorial**.
Complex Numbers	A complex number is formed by adding a pure imaginary number to a real number. The general form of a complex number is $z = x + iy$, where x and y are both real numbers and i is the imaginary unit: $i^2 = -1$. The number x is called the real part of the complex number, while the number y is called the imaginary part of the complex number.
Conditional Probability	The conditional probability is the probability of the occurrence of one event affecting another event. The conditional probability of event A occurring given that even B has occurred is denoted $P(A\|B)$ (read "probability of A given B"). The formula for conditional probability is $P(A\|B) = \frac{P(A \text{ and } B)}{P(B)}$.
Conjugate of a Complex Number	The conjugate of a complex number $z = x + iy$ is the complex number $\bar{z} = x - iy$, found by changing the sign of the imaginary part. For example, if $z = 3 - 4i$, then $\bar{z} = 3 + 4i$.
Continuous	The graph of $y = f(x)$ is continuous at a point a if: 1. $f(a)$ exists, 2. $\lim_{x \to a} f(x)$ exists, and 3. $\lim_{x \to a} f(x) = f(a)$. A function is said to be continuous in an interval if it is continuous at each point in the interval.
Continuous Random Variable	A continuous random variable is a random variable that can take on any real number value within a specified range. For contrast, see **Discrete Random Variable.**
Coterminal	Two angles are said to be coterminal if they have the same initial and terminal arms. For example, $\theta = 30^o$ is coterminal with $\alpha = 390^o$.
Critical Point	A critical point of a function $f(x)$ is the point $P(x, \mathrm{y})$ where the first derivative, $f'(x)$ is zero. See also **stationary points.**
Data	Data (plural of datum) are the facts about something. For example, the

WORDS	MEANING
	height of a building.
Degree	1. The degree is a unit of measure for angles. One degree is $\frac{1}{360}$ of a complete rotation. See also **Radian.** 2. The degree of a polynomial is the highest power of the variable that appears in the polynomial. For example, the polynomial $p(x) = 2 + 3x - x^2 + 7x^3$ has degree 3.
Delta	The Greek capital letter delta, which has the shape of a triangle: Δ, is used to represent "change in". For example Δx represents "change in x".
Dependent Events	In Statistics, two events A and B are said to be dependent if the occurrence of one event affects the probability of the occurrence of the other event. For contrast, see **Independent Events.**
Derivative	The derivative of a function $y = f(x)$ is the rate of change of that function. The notations used for derivative include: $y' = f'(x) = \frac{dy}{dx} = \lim_{\Delta x \to 0} \frac{f(x+\Delta x)-f(x)}{\Delta x}$.
Descriptive Statistics	Descriptive statistics refers to a variety of techniques that allows for general description of the characteristics of the data collected. It also refers to the study of ways to describe data. For example, the mean, median, variance and standard deviation are descriptive statistics. For contrast, see **Inferential Statistics.**
Determinant	The determinant of a matrix is a number that is useful for describing the characteristics of the matrix. For example if the determinant is zero then the matrix has no inverse.
Differentiable	A continuous function is said to be differentiable over an interval if its derivative exists for every point in that interval. That means that the graph of the function is smooth with no kinks, cusps or breaks.
Differential Equation	A differential equation is an equation involving the derivatives of a function of one or more variables. For example, the equation $\frac{dy}{dx} - y = 0$ is a differential equation.
Differentiation	Differentiation is the process of finding the derivative.
Discrete	A set of values are said to be discrete if they are all distinct and separated from each other. For example the set of shoe sizes where the elements of this set can only take on a limited and distinct set of values. See **Discrete Random Variables.**
Discrete Random Variable	A discrete random variable is a random variable that can only take on values from a discrete list. For contrast, see **Continuous Random Variables.**

WORDS	MEANING
Estimate	The best guess for an unknown quantity arrived at after considering all the information given in a problem.
Even Function	A function $y = f(x)$ is said to be even if it satisfies the property that $f(x) = f(-x)$. For example, $f(x) = \cos x$ and $g(x) = x^2$ are even functions. For contrast, see Odd Function.
Event	In probability, an event is a set of outcomes of an experiment. For example, the even A may be defined as obtaining two heads from tossing a coin twice.
Expected Value	The average amount that is predicted if an experiment is repeated many times. The expected value of a random variable X is denoted by $E[X]$. The expected value of a discrete random variable is found by taking the sum of the product of each outcome and its associated probability. In short, $E[X] = \sum_{i=1}^{n} x_i p(x_i)$.
Experimental Probability	Experimental probability is the chances of something happening, based on repeated testing and observing results. It is the ratio of the number of times an event occurred to the number of times tested. For example, to find the experimental probability of winning a game, one must play the game many times, then divide by the number of games won by the total number of games played.
Exponent	An exponent is a symbol or a number written above and to the right of another number. It indicates the operation of repeated multiplication.
Exponential Function	A function that has the form $y = a^x$, where a is any real number and is called the base.
Extrapolation	An extrapolation is a predicted value that is outside the range of previously observed values. For contrast, see **Interpolation**.
Factor	A factor is one of two or more expressions which are multiplied together. A prime factor is an indecomposable factor. For example, the factors of $(x^2 - 4)(x + 3)$ include $(x^2 - 4)$ and $(x + 3)$, where $(x + 3)$ is prime but $(x^2 - 4)$ is not prime as it can be further decomposed into $(x - 2)(x + 2)$.
Factorial	The factorial of a positive integer n is the product of all the integers from 1 up to n and is denoted by $n!$, where $1! = 0! = 1$. For example, $5! = 5 \times 4 \times 3 \times 2 \times 1 = 120$.
Function	A correspondence in which each member of one set is mapped unto a member of another set.

WORDS	**MEANING**
Geometric Progression	A geometric progression is a sequence of terms obtained by multiplying the previous term by a fixed number which is called the common ratio. A geometric progression is of the form a, ar, ar^2, ar^3, ….
Graph	A visual representation of data that displays the relationship among variables, usually cast along x and y axes.
Grouped Data	Grouped data refers to a range of values which are combined together so as to make trends in the data more apparent.
Heterogeneity	Heterogeneity is the state of being of incomparable magnitudes. For contrast, see **Homogeneity.**
Heuristic	A heuristic method of solving problems involve intelligent trial and error. For contrast, see **Algorithm.**
Histogram	A histogram is a bar graph with no spaces between the bars where the area of the bars are proportionate to the corresponding frequencies. If the bars have the same width then the heights are proportionate to the frequencies.
Homogeneity	Homogeneity is the state of being of comparable magnitudes. For contrast, see **Heterogeneity.**
Identity	1. An equation that is true for every possible value of the variables. For example $x^2 - 1 \equiv (x-1)(x+1)$ is an identity while $x^2 - 1 = 3$ is not, as it is only true for the values $x = \pm 2$. 2. The **identity element** of an operation is a number such that when operated on with any other number results in the other number. For example, the identity element under addition of real numbers is zero; the identity element under multiplication of 2×2 matrices is $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$.
Independent Events	In Statistics, two events are said to be independent if they do not affect each other. That is, the occurrence of one event does not depend on whether or not the other event occurred.
Inferential Statistics	Inferential Statistics is the branch of mathematics which deals with the generalisations of samples to the population of values.
Infinity	The symbol ∞ indicating a limitless quantity. For example, the result of a nonzero number divided by zero is infinity.
Integration	Integration is the process of finding the integral which is the antiderivative of a function.

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Interpolation	An interpolation is an estimate of an unknown value which is within the range of previously observed values. For contrast, see **Extrapolation.**
Interval	An interval on a number line is a continuum of points bounded by two limits (end points). An **Open Interval** refers to an interval that excludes the end points and is denoted (a, b). For example, $(0,1)$. A **Closed Interval** in an interval which includes the end points and is denoted $[a, b]$. For example $[-1,3]$. A **Half-Open Interval** is an interval which includes one end point and excludes the other. For example, $[0, \infty)$.
Interval Scale	Interval scale refers to data where the difference between values can be quantified in absolute terms and any zero value is arbitrary. Finding a ratio of data values on this scale gives meaningless results. For example, temperature is measured on the interval scale: the difference between 19^oC and 38^oC is 19^oC, however, 38^oC is not twice as warm as 19^oC and a temperature of 0^oC does not mean there is no temperature. See also Nominal, Ordinal and Ratio scales.
Inverse	1. **The inverse of an element under an operation** is another element which when operated on with the first element results in the identity. For example, the inverse of a real number under addition is the negative of that number. 2. **The inverse of a function** $f(x)$ is another function denoted $f^{-1}(x)$, which is such that $f[f^{-1}(x)] = f^{-1}[f(x)] = x$.
Irrational Number	A number that cannot be represented as an exact ratio of two integers. For example, π or the square root of 2.
Limit	The limit of a function is the value which the dependent variable approaches as the independent variable approaches some fixed value.
Line of Best Fit	The line of best fit is the line that minimises the sum of the squares of the deviations between each point and the line.
Linear Expression	An expression of the form $ax + b$ where x is a variable and a and b are constants, or in more variables, an expression of the form $ax + by + c, ax + by + cz + d$ where a, b, c and d are constants.
Logarithm	A logarithm is the power of another number called the base that is required to make its value a third number. For example 3 is the logarithm which carries 2 to 8. In general, if y is the logarithm which carries a to x, then it is written as $y = \log_a x$ where a is called the base. There are two popular bases: base 10 and base e. 1. **The Common Logarithm (Log)**: the equation $y = \log x$ is the shortened form for $y = \log_{10} x$.

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	2. **The Natural Logarithm (Ln)**: The equation $y = \ln x$ is the shortened form for $y = \log_e x$
Matrix	A rectangular arrangement of numbers in rows and columns.
Method	In Statistics, the research methods are the tools, techniques or processes that we use in our research. These might be, for example, surveys, interviews, or participant observation. Methods and how they are used are shaped by methodology.
Methodology	Methodology is the study of how research is done, how we find out about things, and how knowledge is gained. In other words, methodology is about the principles that guide our research practices. Methodology therefore explains why we're using certain methods or tools in our research.
Modulus	The modulus of a complex number $z = x + iy$ is the real number $\lvert z \rvert = \sqrt{x^2 + y^2}$. For example, the modulus of $z = -7 + 24i$ is $\lvert z \rvert = \sqrt{(-7)^2 + 24^2} = 25$
Mutually Exclusive Events	Two events are said to be mutually exclusive if they cannot occur simultaneously, in other words, if they have nothing in common. For example, the event "Head" is mutually exclusive to the event "Tail" when a coin is tossed.
Mutually Exhaustive Events	Two events are said to be mutually exhaustive if their union represents the sample space.
Nominal Scale	Nominal scale refers to data which names of the outcome of an experiment. For example, the country of origin of the members of the West Indies cricket team. See also Ordinal, Interval and Ratio scales.
Normal	The normal to a curve is a line which is perpendicular to the tangent to the curve at the point of contact.
Odd Function	A function is an odd function if it satisfies the property that $f(-x) = -f(x)$. For example, $f(x) = \sin x$ and $g(x) = x^3$ are odd functions. For contrast, see **Even Function**.
Ordinal Scale	Data is said be in the ordinal scale if they are names of outcomes where sequential values are assigned to each name. For example, if Daniel is ranked number 3 on the most prolific goal scorer at the Football World Cup, then it indicates that two other players scored more goals than Daniel. However, the difference between the 3rd ranked and the 10th ranked is not necessarily the same as the difference between the 23rd and 30th ranked players. See also **Nominal, Interval and Ratio scales**.
Outlier	An outlier is an observed value that is significantly different from the other observed values.

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Parameter	In statistics, a parameter is a value that characterises a population.
Partial Derivative	The partial derivative of $y = f(x_1, x_2, x_3, \dots, x_n)$ with respect to x_i is the derivative of y with respect to x, while all other independent variables are treated as constants. The patrial derivative is denoted by $\frac{\partial f}{\partial x}$. For example, if $f(x, y, z) = 2xy + x^2z - \frac{3x^3y}{z}$, then $\frac{\partial f}{\partial x} = 2y + 2xz - \frac{9x^2y}{z}$
Pascal Triangle	The Pascal triangle is a triangular array of numbers such that each number is the sum of the two numbers above it (one left and one right). The numbers in the n^{th} row of the triangle are the coefficients of the binomial expansion $(x + y)^n$.
Percentile	The p^{th} percentile of in a list of numbers is the smallest value such that p% of the numbers in the list is below that value. See also **Quartiles.**
Permutations	Permutations refers to the number of different ways of selecting a group of r objects from a set of n object when the order of the elements in the group is of importance and the items are not replaced. If $r = n$ then the permutations is $n!$; if $r < n$ then the number of permutation is $P_r^n = \frac{n!}{(n-r)!}$.
Piecewise Continuous Function	A function is said to be piecewise continuous if it can be broken into different segments where each segment is continuous.
Polynomial	A polynomial is an algebraic expression involving a sum of algebraic terms with nonnegative integer powers. For example, $2x^3 + 3x^2 - x + 6$ is a polynomial in one variable.
Population	In statistics, a population is the set of all items under consideration.
Principal Root	The principal root of a number is the positive root. For example, the principal square root of 36 is 6 (not -6).
Principal Value	The principal value of the arcsin and arctan functions lies on the interval $-\frac{\pi}{2} \leq x \leq \frac{\pi}{2}$. The principal value of the arcos function lies on the interval $0 \leq x \leq \pi$.
Probability	1. The probability of an event is a measure of how likely it is for an event to occur. The probability of an event is always a number between zero and 1. 2. Probability is the study of chance occurrences.
Probability Distribution	A probability distribution is a table or function that gives all the possible values of a random variable together with their respective probabilities.

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Probability Space	The probability space is the set of all outcomes of a probability experiment.
Proportion	1. A relationship between two ratios in which the first ratio is always equal to the second. Usually of the form $\frac{a}{b} = \frac{c}{d}$. 2. The fraction of a part and the whole. If two parts of a whole are in the ratio 2:7, then the corresponding proportions are $\frac{2}{9} \text{and} \frac{7}{9}$ respectively.
Pythagorean Triple	A Pythagorean triple refers to three numbers, a, b & c, satisfying the property that $a^2 + b^2 = c^2$.
Quadrant	The four parts of the coordinate plane divided by the x and y axes are called quadrants. Each of these quadrants has a number designation. First quadrant – contains all the points with positive x and positive y coordinates. Second quadrant – contains all the points with negative x and positive y coordinates. The third quadrant contains all the points with both coordinates negative. Fourth quadrant – contains all the points with positive x and negative y coordinates.
Quadrantal Angles	Quadrantal Angles are the angles measuring $0^{\mathrm{o}}, 90^{\mathrm{o}}, 180^{\mathrm{o}}$ & 270^{o} and all angles coterminal with these. See **Coterminal.**
Quartic	A quartic equation is a polynomial of degree 4.
Quartiles	Consider a set of numbers arranged in ascending or descending order. The quartiles are the three numbers which divide the set into four parts of equal amount of numbers. The first quartile in a list of numbers is the number such that a quarter of the numbers is below it. The second quartile is the median. The third quartile is the number such that three quarters of the numbers are below it. See also **Percentile.**
Quintic	A quintic equation is a polynomial of degree 5.
Radian	The radian is a unit of measure for angles, where one radian is $\frac{1}{2\pi}$ of a complete rotation. One radian is the angle in a circle subtended by an arc of length equal to that of the radius of the circle. See also **Degrees**.
Radical	The radical symbol ($\sqrt{}$) is used to indicate the taking of a root of a number. $\sqrt[q]{x}$ means the q^{th} root of x; if $q = 2$ then it is usually written as $\sqrt{x}$. For example $\sqrt[5]{243} = 3$, $\sqrt[4]{16} = 2$. The radical always means to take the positive value. For example, both 5 and -5 satisfy the equation $\mathrm{x}^2 = 25$, but $\sqrt{25} = 5$.
Random Variable	A random variable is a variable that takes on a particular value when a random event occurs.

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Ratio Scale	Data are said to be on the ratio scale if they can be ranked, the distance between two values can be measured and the zero is absolute, that is, zero means "absence of". See also Nominal, Ordinal and Interval Scales.
Regression	Regression is a statistical technique used for determining the relationship between two quantities.
Residual	In linear regression, the residual refers to the difference between the actual point and the point predicted by the regression line. That is the vertical distance between the two points.
Root	1. The root of an equation is the same as the solution of that equation. For example, if $y = f(x)$, then the roots are the values of x for which $y = 0$. Graphically, the roots are the x-intercepts of the graph. 2. The n^{th} root of a real number x is a number which, when multiplied by itself n times, gives x. If n is odd then there is one root for every value of x; if n is even then there are two roots (one positive and one negative) for positive values of x and no real roots for negative values of x. The positive root is called the **Principal root** and is represented by the radical sign ($\sqrt{}$). For example, the principal square root of 9 is written as $\sqrt{9} = 3$ but the square roots of 9 are $\pm\sqrt{9} = \pm 3$.
Sample	A group of items chosen from a population.
Sample Space	The set of all possible outcomes of a probability experiment. Also called probability space.
Sampling Frame	In statistics, the sampling frame refers to the list of cases from which a sample is to be taken.
Scientific Notation	A shorthand way of writing very large or very small numbers. A number expressed in scientific notation is expressed as a decimal number between 1 and 10 multiplied by a power of 10 (for example, 7000 = $7\text{x}10^3$ or 0.0000019 = $1.9\text{x}10^{-6}$).
Series	A series is an indicated sum of a sequence.
Sigma	1. The Greek capital letter sigma, Σ, denotes the summation of a set of values. 2. The corresponding lowercase letter sigma, σ, denotes the standard deviation.
Significant Digits	The amount of digits required for calculations or measurements to be close enough to the actual value. Some rules in determining the number of digits considered significant in a number:

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	- The leftmost non-zero digit is the first significant digit. - Zeros between two non-zero digits are significant. - Trailing zeros to the right of the decimal point are considered significant.
Simple Event	A non-decomposable outcome of a probability experiment.
Skew	Skewness is a measure of the asymmetry of a distribution of data.
Square Matrix	A matrix with equal number of rows and columns.
Square Root	The square root of a positive real number n is the number m such that $m^2 = n$. For example, the square roots of 16 are 4 and -4.
Standard Deviation	The standard deviation of a set of numbers is a measure of the average deviation of the set of numbers from their mean.
Stationary Point	The stationary point of a function $f(x)$ is the point $P(x_o, y_o)$ where $f'(x) = 0$. There are three type of stationary points, these are: 1. Maximum point is the stationary point such that $\frac{\mathrm{d}^2 f}{\mathrm{d}x^2} \leq 0$; 2. Minimum point is the stationary point such that $\frac{\mathrm{d}^2 f}{\mathrm{d}x^2} \geq 0$; 3. Point of Inflexion is the stationary point where $\frac{\mathrm{d}^2 f}{\mathrm{d}x^2} = 0$ and the point is neither a maximum nor a minimum point.
Statistic	A statistic is a quantity calculated from among the set of items in a sample.
Statistical Inference	The process of estimating unobservable characteristics of a population by using information obtained from a sample.
Symmetry	Two points A and B are symmetric with respect to a line if the line is a perpendicular bisector of the segment AB.
Tangent	A line is a tangent to a curve at a point A if it just touches the curve at A in such a way that it remains on one side of the curve at A. A tangent to a circle intersects the circle only once.
Theoretical Probability	The chances of events happening as determined by calculating results that would occur under ideal circumstances. For example, the theoretical probability of rolling a 4 on a fair four-sided die is ¼ or 25%, because there is one chance in four to roll a 4, and under ideal circumstances one out of every four rolls would be a 4.
Trigonometry	The study of triangles. Three trigonometric functions defined for either acute angles in the right-angled triangle are: Sine of the angle x is the ratio of the side opposite the angle and the

WORDS	MEANING
	hypotenuse. In short, $\sin x = \frac{O}{H}$; Cosine of the angle x is the ratio of the short side adjacent to the angle and the hypotenuse. In short, $\cos x = \frac{A}{H}$; Tangent of the angle x is the ratio of the side opposite the angle and the short side adjacent to the angle. In short $\tan x = \frac{O}{A}$.
Z-Score	The z-score of a value x is the number of standard deviations it is away from the mean of the set of all values. $z - score = \frac{x-\bar{x}}{\sigma}$.

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SPECIMEN PAPER

PAPER 01

1 hour 30 minutes

READ THE FOLLOWING INSTRUCTIONS CAREFULLY.

1. This specimen paper consists of 45 items. You will have 1 hour and 30 minutes to answer them.

2. In addition to the test booklet, you should have an answer sheet.

3. Each item in this test has four suggested answers lettered (A), (B), (C), (D). Read each item you are about to answer and decide which choice is best.

4. Find the number which corresponds to your item and shade the space having the same letter as the answer you have chosen. Look at the sample item below.

   Sample Item

   Which of the following is equivalent to $y = e^{2x}$?

   (A) $y = 2e^x$
   (B) $\log y = 2x$
   (C) $\ln 2x = y$
   (D) $\ln y = 2x$

   Sample Answer

   Ⓐ Ⓑ Ⓒ ●

   The best answer to this item is "Ln $y = 2x$", so (D) has been shaded.

5. You may use silent, non-programmable calculators to answer questions.

DO NOT TURN THIS PAGE UNTIL YOU ARE TOLD TO DO SO.

1. The conjugate of $-2 - 3i$ is

(A) $-2 + 3i$
(B) $2 - 3i$
(C) $-3 - 2i$
(D) $\sqrt{13}$

2. Which of the following represents the complex solution to the equation $2x^2 + 3x + 4 = 0$

(A) $\frac{-3 \pm i\sqrt{19}}{4}$
(B) $\frac{-3 \pm i\sqrt{41}}{4}$
(C) $\frac{-3 \pm \sqrt{23}}{4}$
(D) $\frac{-3 \pm i\sqrt{23}}{4}$

3. Find the equation of the line passing through the midpoint of (2, 6) and (–8, 10) with gradient 2.

(A) $5y + 2x = 34$
(B) $y + 7 = 2x$
(C) $y = 2x + 14$
(D) $2y = 5x + 31$

4. The gradient of the straight line that makes an angle of 30^o with the x-axis is

(A) sin 30°
(B) cos 30°
(C) tan 30°
(D) tan 60°

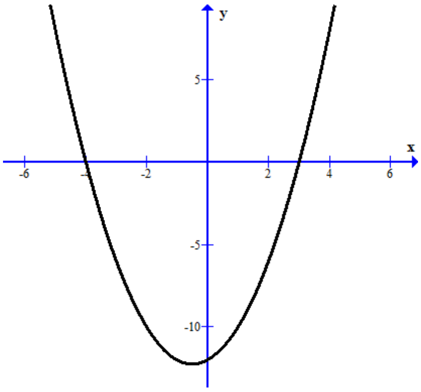
5. Which of the following points satisfies both of the equations below?

$2y = x - 7,$
$\frac{9}{x} + \frac{4}{y} = 1$

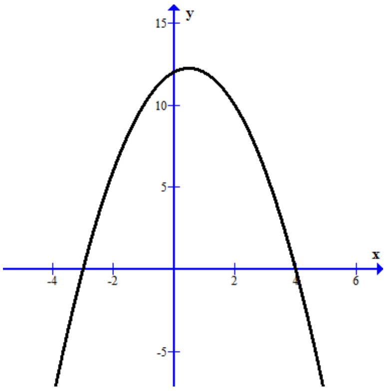
(A) (3, -2)
(B) (-3, 2)
(C) (-3, -2)
(D) (3, 2)

6. Which of the following BEST represents the graph of $y = x^2 + x - 12$?

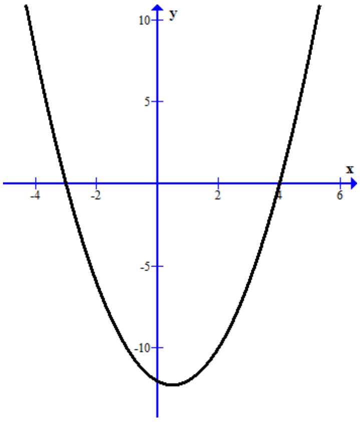
(A)



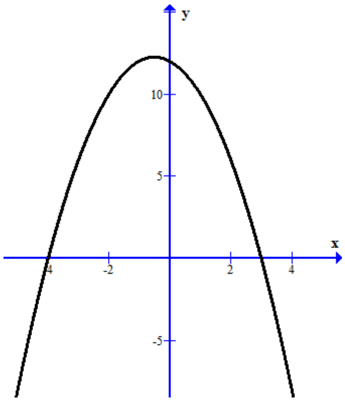
(B)



(C)



(D)



7. If $f(x) = x^2$ and h(x) = $2 - x$, then hf(–3) =

(A) –1
(B) 11
(C) –7
(D) 25

8. log 16 $-$ log 2 is the same as

(A) $\frac{\log 16}{\log 2}$
(B) log8
(C) log (14)
(D) log 16^2

9. If log $\sqrt[3]{y} = \frac{1}{3}$, then y is equal to

(A) $\sqrt[3]{9}$
(B) 1
(C) $\left(\frac{1}{3}\right)^3$
(D) 10

10. If $9^x = \frac{1}{27}$, then x is equal to

(A) –3
(B) –1.5
(C) 1.5
(D) 3

11. If f $(x) = c + 2x - x^3$ and $f(-1) = 5$, then the value of c is

(A) 4
(B) 5
(C) 6
(D) 8

12. The n^{th} term in the arithmetic progression 5, 11, 17, 23, 29 is

(A) $6n - 1$
(B) $5n + 6$
(C) $n - 6$
(D) $6n + 11$

13. If $|3\text{x} - 9| \geq 12,$ which of the following is NOT a possible value of x?

(A) $x = -18$
(B) $x = 1$
(C) $x = 7$
(D) $x = 21$

14. What is the sum, to infinity, of the geometric progression $1 - \frac{1}{3} + \frac{1}{9} - \frac{1}{27} + \cdots$?

(A) $\frac{3}{4}$
(B) $\frac{20}{27}$
(C) $\frac{3}{2}$
(D) 0

15. If $\begin{pmatrix} -2 & 0 \\ 3 & -3 \end{pmatrix} + \begin{pmatrix} 4 & 1 \\ 0 & 7 \end{pmatrix} = \begin{pmatrix} 2 & 1 \\ 3 & x^2 \end{pmatrix}$, then
which of the following is a possible value of x?

(A) –3
(B) –2
(C) 4
(D) 7

Items **16–18** refer to the information in the following table, which shows the National Test Summary Scores of the students at two schools: Gibbons and Rawlins.

	Schools	
	Gibbons	**Rawlins**
Mean	68	43
Median	61	45
Range	35	28
Standard Deviation	9	6
Number of Students	97	104

16. Which value, in the table, is MOST suitable for determining the school with the more homogeneous national test scores?

(A) Mean
(B) Range
(C) Median
(D) Standard deviation

17. What can be deduced about the shape of the distribution curve for the national test scores at Gibbons?

(A) Negatively skewed
(B) Positively skewed
(C) Normal
(D) Symmetrical

18. If the pass mark was 45, then which of the following most likely represents the number of the students from Rawlins that passed the test?

(A) 43
(B) 45
(C) 50
(D) 52

Items **19─20** refer to the following scenario.

In preparation for his school exams, Ray set aside four one-hour segments daily to study four different subjects. He has a total of six subjects to study and each hour is assigned to exactly one subject. No subject is studied for more than an hour on a given day.

19. How many groups of four subjects from among the six can he select to study on a given day?

(A) 1
(B) 3
(C) 15
(D) 24

20. Mathematics and History were selected with two other subjects to be studied on Monday. What is the probability that Mathematics and History are studied during the first two hours on Monday?

(A) $\frac{1}{16}$

(B) $\frac{1}{12}$

(C) $\frac{1}{6}$

(D) $\frac{1}{2}$

21. The heights of 400 adults were measured and recorded. If the heights collected were subsequently verified to be normally distributed, then which of the following statements MUST be true?

(A) A positive *z*-score corresponds to a value greater than the median.
(B) The heights of exactly 68% of the adults are within one standard deviation of the mean.
(C) The heights obtained are positively skewed.
(D) The variance is small.

22. The time it takes for a bus to complete the route from bus terminal A to bus terminal B follows a normal distribution with mean 25 minutes and standard deviation 5 minutes. Given that the z-score of a particular bus on one such trip is $z = 1.731$, what is the time taken for that trip?

(A) 0.958
(B) 26.731
(C) 33.655
(D) 68.275

23. The Star brand of 20-watt light bulbs has a 0.2 probability of not lasting the estimated life of three years of daily use. An office building in Kingston installed 60 of these bulbs on 31January 2014. How many of these bulbs are expected to light after 31January 2017, that is after three years of daily use?

(A) 0.8
(B) 12.0
(C) 48.0
(D) 60.0

<u>Items **24–25**</u> refer to the following table which shows the number and brand of cars in three cities.

	Brand of Car			
City	**Toyota**	**Nissan**	**Other**	**Total**
A	30	50	20	100
B	100	60	90	250
C	40	80	30	150
Total	170	190	140	500

24. Given that a car is selected at random from city B, what is the probability that it is a Nissan?

(A) 0.12
(B) 0.24
(C) 0.32
(D) 0.60

25. A car is randomly selected from among the three cities. What is the probability that it is from city C but is neither a Toyota nor a Nissan?

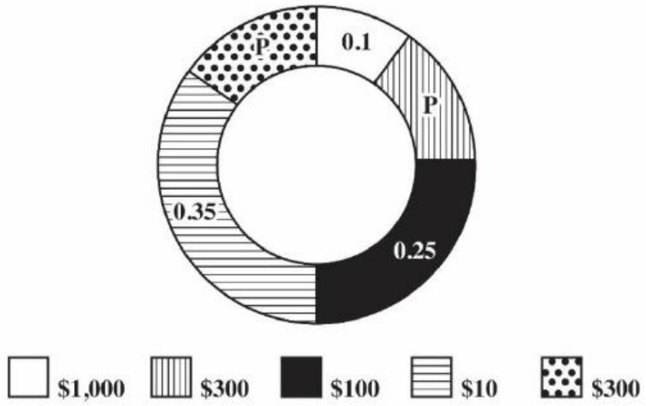
(A) 0.056
(B) 0.060
(C) 0.200
(D) 0.280

26. Data were collected from 100 persons when they joined a gym. Given that the correlation coefficient for their weights and heights is equal to 0.86, which of the following statements is true about the relationship between their weights and heights?

(A) The correlation coefficient is insufficient to determine a relationship between their weights and heights.
(B) There is only a small fraction of relation between their weights and heights.
(C) There is a high positive correlation between their weights and heights.
(D) There is correlation among the weights of 86 of these persons.

<u>Items **27–29**</u> refer to the game described below.

A game at a fair consists of spinning a vertically erected wheel in order to win a monetary prize. The following diagram shows the wheel with the probability of winning each section displayed on the wheel and the amount of prize money to be won in each section is displayed in the legend. For example, the probability of winning $100.00 is 0.25. The wheel has a small ball such that a player wins the money labelled on the section in which the ball rests when the wheel comes to a stop. The position on the wheel where the ball comes to rest is random.



27. If $2p$ represents the probability of winning $300, what is the value of p?

(A) 0.125
(B) 0.15
(C) 0.2
(D) 0.3

28. Which of the following are the expected gains (expected value) of playing the game?

(A) $218.50
(B) $308.50
(C) $1000.00
(D) $1710.00

29. Judah played the game eight times. What is the probability that he won $1000.00 on exactly two of those eight trials? [Hint: binomial distribution]

(A) 0.10
(B) 0.15
(C) 0.20
(D) 0.80

30. The height of a toddler is measured and recorded every three months for four years. Which of the following techniques is most appropriate for representing the toddler's height over the four-year period?

(A) A bar graph
(B) A histogram
(C) A stem-and-leaf plot
(D) A line graph

31. $\lim_{x \to -3} \frac{x^2 - 9}{5} =$

(A) $-\frac{18}{5}$
(B) 0
(C) $\frac{18}{5}$
(D) ∞

<u>Items **32–33**</u> refer to the piecewise function $f(x) = \begin{cases} 4x - 1 & : x < -3 \\ 3 + x & : x \geq -3 \end{cases}$

32. $\lim_{x \to -3^+} f(x) =$

(A) –13
(B) 0
(C) 6
(D) 11

33. For what value of x is $f(x)$ discontinuous?

(A) 0
(B) no value of x
(C) 1
(D) –3

34. The gradient of the function $f(x) = 2x^3 - 3x + 5$ at $x = 1$ is

(A) –3
(B) 0
(C) 3
(D) 5

35. Which of the following rules should be used to differentiate $f(x) = \frac{3x+1}{2-x^2}$?

(A) Chain rule
(B) Power rule
(C) Quotient rule
(D) Logarithm rule

36. A firm's profit function is given by $P(x) = -0.25x^2 + 3x - 5$.
For what value of x is the marginal profit, $P'(x)$, equal to zero?

(A) –5
(B) 2
(C) 3
(D) 6

37. $\int(4x - 4)dx =$

(A) $2x^2 + 4x + C$
(B) $2x^2 - 4 + C$
(C) $2x^2 - 4x + C$
(D) 4

38. If $\frac{dy}{dx} = x^2 - 4x - 5$, then the critical points of the function y are at $x =$

(A) 1 and –5
(B) –1 and 5
(C) –1 and –5
(D) 1 and 5

39. The critical values of the function $g(x) = x^3 - 27x$ are $x = -3$ and $x = 3$. What is the maximum value?

(A) –18
(B) 18
(C) 54
(D) 108

40. If $h(x) = \sin 5x$, then $h'(x) =$

(A) $\frac{1}{5}\cos 5x$
(B) $-\frac{1}{5}\cos 5x$
(C) 5 cos x
(D) 5 cos 5x

41. Evaluate $\lim_{x \to 2} \frac{x^2 - 4}{x - 2}$

(A) 0
(B) ∞
(C) 4
(D) 2

42. $\int cos(\pi - 3x)dx =$

(A) $-3sin(\pi - 3x) + C$

(B) $\frac{1}{3}sin(\pi - 3x) + C$

(C) $3sin(\pi - 3x) + C$

(D) $-\frac{1}{3}sin(\pi - 3x) + C$

43. $\int \left(\frac{4}{x}\right) dx =$

(A) $\frac{8}{x^2} + C$

(B) $\frac{x^5}{5} + C$

(C) $4 \ln |x| + C$

(D) $\ln |x| + C$

44. Given that $y = e^x \ln x$, what is the derivative of *y*?

(A) $e^x \left(\frac{1}{x} + \ln x\right)$

(B) $e^x \frac{1}{x}$

(C) $e^x \ln x$

(D) $e^x (1 + \ln x)$

45. The function $f(x) = 2x^3 - 3x + 5$ at at the point where $x = 0$ is

(A) decreasing
(B) increasing
(C) negative
(D) not defined

END OF TEST

CAPE INTEGRATED MATHEMATICS

PAPER 1

KEY

QUESTION NUMBER	KEY	SO	SKILL
1	A	MO 1.1.2	CK
2	D	MO 1.1.4	AK
3	C	MO 1.2.1	AK
4	C	MO 1.2.2	CK
5	A	MO 1.3.2	AK
6	A	MO 1.3.4	AK
7	C	MO 1.3.7	AK
8	B	MO 1.4.3	CK
9	D	MO 1.4.4	R
10	B	MO 1.4.7	AK
11	C	MO 1.5.1	AK
12	A	MO 1.6.3	CK
13	B	MO 1.3.5	AK
14	A	MO 1.6.4	AK
15	B	MO 1.7.1	R
16	D	MO 2.3.5	AK
17	B	MO 2.2.3	R
18	D	MO 2.5.14	AK
19	C	MO 2.4.3	AK
20	C	MO 2.4.2	AK
21	A	MO 2.5.7	R
22	C	MO 2.5.8	AK
23	C	MO 2.5.4	AK
24	B	MO 2.5.2	AK
25	B	MO 2.5.2	AK
26	C	MO 2.5.9	CK
27	B	MO 2.5.5	CK
28	A	MO 2.5.5	AK
29	B	MO 2.5.6	CK
30	D	MO 2.2.2	CK
31	B	MO 3.1.3	CK
32	B	MO 3.1.5	AK
33	D	MO 3.1.6	AK
34	C	MO 3.2.3	AK
35	C	MO 3.2.7	CK
36	D	MO 3.2.7	R
37	C	MO 3.4.2	AK
38	B	MO 3.3.4	AK
39	C	MO 3.3.5	AK
40	D	MO 3.2.4	AK
41	C	MO 3.1.4	AK
42	D	MO 3.4.3	R
43	C	MO 3.4.4	CK
44	A	MO 3.2.7	AK
45	A	MO 3.3.3	CK

TEST CODE **02167020**

SPEC 2015/02167020

CARIBBEAN EXAMINATIONS COUNCIL

CARIBBEAN ADVANCED PROFICIENCY EXAMINATION®

INTEGRATED MATHEMATICS

PAPER 02

SPECIMEN PAPER

2 hours 30 minutes

READ THE FOLLOWING INSTRUCTIONS CAREFULLY

1. This paper consists of THREE sections. Each section consists of 2 questions.
2. Answer ALL questions from the THREE sections.
3. Unless otherwise stated in the question, any numerical answer that is not exact MUST be written correct to THREE significant figures.

Examination Materials Permitted
Mathematical formulae and tables (provided)
Silent, non programmable Electronic calculator
Mathematical Instruments

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SECTION A
MODULE 1: FOUNDATIONS OF MATHEMATICS

Answer BOTH questions.

1. (a) Given that x and y are real numbers, find the value of x and the value of y if $(x+y)+(x-y)i=7+3i$.

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(3 marks)

(b) The line $y=x$ and the curve $y=4x-x^2$ intersect at the points P and Q.

(i) Find the coordinates of P and Q.

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(3 marks)

(ii) Find the equation of the perpendicular bisector of PQ.

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(4 marks)

(c) (i) Obtain the expansion for $(2+x)^3$

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(3 marks)

(ii) Using the result from c (i) or otherwise, obtain an expansion for $(2-3x)^3$.

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(2 marks)

(iii) Hence, obtain the expression for $(2+x)^3-(2-3x)^3$.

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(2 marks)

(d) Find the range of values of x for which $2x^2+x-15\geq 0$.

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(3 marks)

(e) Determine the value of $\frac{\pi}{6} - \sin\left(\frac{\pi}{6}\right)$.

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(2 marks)

(f) Solve the equation $\cos\left(\theta + \frac{3\pi}{4}\right) = 0.5$, for the values of θ where

$-\pi \leq \theta \leq \pi$.

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(3 marks)

Total 25 marks

2. (a) Solve the following equations for x

(i) $3^{x-1} \times 27^{3x-1} = 81^{2x}$.

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(3 marks)

(ii) $\log(x) + \log(x-3) = 1$.

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(4 marks)

(b) (i) Find the value of b given that the polynomial $f(x) = x^3 + bx^2 - 4x - 5$ when divided by $x - 3$ has a remainder of -8.

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(3 marks)

(ii) Given that $g(x) = 2x^3 + 7x^2 + x - 10$, factorize g ($x$) completely.

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(4 marks)

(c) The eighth term of an arithmetic progression is 150 and the fifty-third term is –30.

Determine the first term and the common difference.

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(4 marks)

(d) (i) Express the following system of equations in the matrix form $AX = b$

$$x + y + z = 4$$
$$2x - 3y = 7$$
$$4x + y - 2z = 1$$

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(2 marks)

(ii) Given that the determinant of A, $|A| = 24$, use Cramer's rule, or otherwise, to solve the system of equations in (d) (i) above.

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(5 marks)

Total 25 marks

SECTION B

MODULE 2: STATISTICS

Answer BOTH questions.

3. The nutrition teacher of a two-year college for adults complained to the principal that the eating habits of the Year 1 students were unhealthy, and as a result, the college should introduce a mandatory nutrition course for the next academic year. One hundred students enroll at the college each year. The principal collected data, as shown in the table below, on the masses in pounds of 15 students in Year 1 and 15 students in Year 2.

Year 1	165	155	170	165	160	155	160	130	160	160	170	160	155	170	180
Year 2	165	145	150	155	130	165	150	145	150	140	155	150	150	200	165

(a) Other than the variable mass, identify ONE other variable for which useful data can be collected to help the principal in making the decision about introducing the nutrition class.

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(1 mark)

(b) The principal measured the masses of a sample of 30 students after first deciding to take all 15 students in the Year 1 French class and similarly all 15 students in the Year 2 Computer class. Give TWO reasons why this sampling method is flawed.

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(3 marks)

(c) Describe a more appropriate process for selecting the sample of 30 students from the college.

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(3 marks)

(d) Advise the principal on a suitable method for graphically representing the data collected on the masses of students, and give a reason for your selection.

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(2 marks)

(e) Calculate the mean and the standard deviation mass of the students in Year 1.

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(5 marks)

(f) Given that the mean and standard deviation masses of the students in Year 2 are 154.3 lbs and 16 lbs respectively, determine which year group has the more homogenous (less variable) weights. Give a reason for your answer.

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(3 marks)

(g) The principal plans to interview students from the sample of 30.

(i) What is the probability of randomly selecting a student with mass at least 170 lbs?

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(3 marks)

(ii) How many groups of 5 students can be selected for interview from among the 30 students if no restrictions are imposed?

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(3 marks)

(h) From the information obtained from Parts (a) to (g), what advice will you give the principal about introducing the nutrition class? Give ONE reason for your answer.

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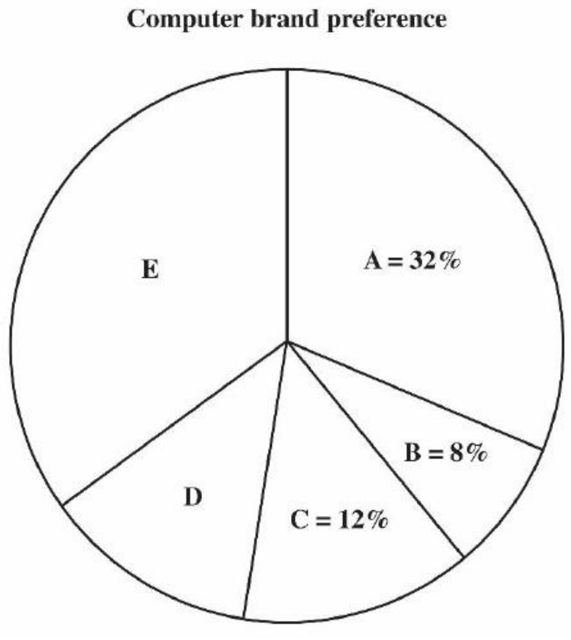
(2 marks)

Total 25 marks

GO ON TO THE NEXT PAGE

4. (a) A business survey was carried out using 750 adults in order to determine preference for computer brands.

The pie chart below summarizes the results of the survey and shows that brand A is preferred by 32% of the adults.



Given that the preference for brand E is three times that of brand D, find the number of persons who preferred brand D

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(3 marks)

(b) A class of 18 teenagers was asked to give the length of time, on an average day, used for family time. The results, in hours, are

2.0	1.4	3.5	3.0	0.4	2.3	1.7	0.0	5.5
1.8	0.5	1.25	2.6	3.1	2.0	0.25	1.9	0.6

(i) Determine the median and quartile times.

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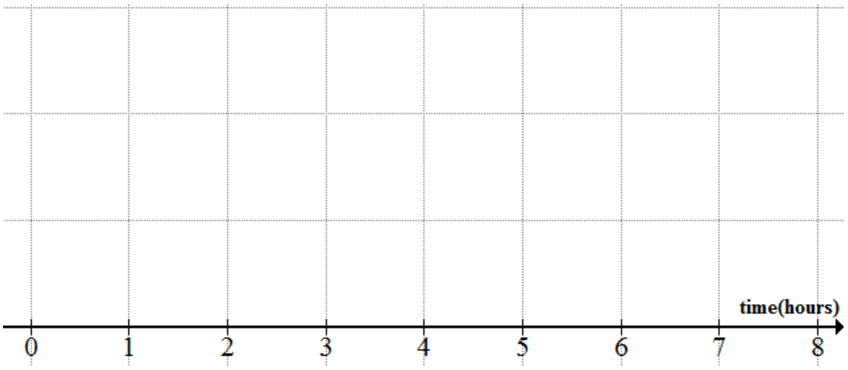
(3 marks)

(ii) Construct a box-and-whiskers plot for the daily time spent as family time.



(3 marks)

(c) Given that a random variable $X \sim Bin(n, p)$ where the mean and variance are 3.6 and 2.16 respectively,

(i) show that $n = 9$ and $p = 0.4$

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(3 marks)

(ii) find $P(X \geq 2)$

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(3 marks)

(d) A farmer measured and labelled the weights of bunches of plantains to be sold on a Friday. Given that the weights of all bunches of plantains on his farm are normally distributed with mean and standard deviation 21 kg and 5 kg respectively, find the probability that a bunch of plantains selected at random weighs 32 kg or more.

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(4 marks)

(e) Company XYZ investigated the possible relation between the distances in km that 40 workers travel from their homes and the number of minutes they are late for work within a given month. The following linear regression equation obtained is:

$$\hat{y} = 5 + 0.4x,$$

where y was treated as the dependent variable minutes late and x as the independent variable distance travelled to work.

(i) Use the regression equation to estimate the number of minutes a worker, who lives 15 km away, is late in a given month.

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(3 marks)

(ii) Comment on the accuracy of your answer to (e) (i.) given that another statistic, the correlation coefficient, was found to be $r = 0.25$.

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(3 marks)

Total 25 marks

SECTION C

MODULE 3: CALCULUS

Answer BOTH questions.

5. (a) Let $f(x)$ be a function defined as $f(x) = \begin{cases} 3 - 2x & \text{for } x > 2 \\ x - 1 & \text{for } x \leq 2 \end{cases}$.

(i) Evaluate $\lim_{x \to 2^-} f(x)$.

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(1 mark)

(ii) Evaluate $\lim_{x \to 2^+} f(x)$.

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(1 mark)

(iii) Is $f(x)$ continuous at $x = 2$? Give a reason for your answer.

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(2 marks)

(b) Evaluate $\lim_{x \to 2} \frac{x-2}{x^2-x-2}$.

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(3 marks)

(c) Differentiate the following with respect to x

(i) $y = \frac{e^{3x}}{5x+\sin x}$

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(4 marks)

(ii) $y = (4x^2 - \ln x)^{1/2}$

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(3 marks)

(d) Find and classify the critical points of the function $g(x) = x^3 - 6x^2 + 9x$.

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(6 marks)

(e) The capacity of expandable right cylindrical tanks is given by $V(r, h) = \pi r^2 h$. At an instant during expansion the radius, r, is 14 cm and the height, h, is 28 cm.

(i) Evaluate the partial derivative $\frac{\partial V}{\partial r}$, the rate of change of V with respect to r.

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(2 marks)

(ii) Give an interpretation for the result.

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[1 Litre = 1000cm^3] **(1 mark)**

(f) A company's total revenue function (**in millions of dollars**) is given by

$$R(x) = -0.008x^3 + 0.24x^2 + 0.6x$$

where x represents the level, in thousands, of demand for its product.

(i) Write an expression for the marginal revenue function $R'(x)$.

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(1 mark)

(ii) Hence, evaluate the marginal revenue when the demand is eight thousand, that is $x = 8$.

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(1 mark)

Total 25 marks

6. (a) (i) Determine $\int (2x-4)^7 \ dx$.

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(3 marks)

(ii) Determine $\int 12x^3 - 2x^2 - 5\,dx$.

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(2 marks)

(b) Evaluate $\int_0^{\pi} (2\cos x + 3\sin x) \ dx$.

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(3 marks)

(c) The rate of increase of the population, *P,* of a parish is proportional to the population size (where *t* is the number of years after 2005).

If in 2005 the population was 1400 and in 2015 it was 2100. Show that $P(t) = 1400e^{\frac{1}{10}ln\frac{3}{2}t}$.

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(6 marks)

(d) In 2014 a company's net income was 16 million dollars. The rate of change of the net income, *N*, since 2014 is known to be $N'(x) = \frac{6}{x+0.5}$ million dollars per year, where x is the number of years since 2014, $x \leq 4$.

(i) Show that the expression for the net income, $N(x)$ is given by

$$N(x) = 6\ln(x + 0.5) + 20.2$$

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(3 marks)

(ii) Calculate the company's income in 2018, that is, when $x = 4$.

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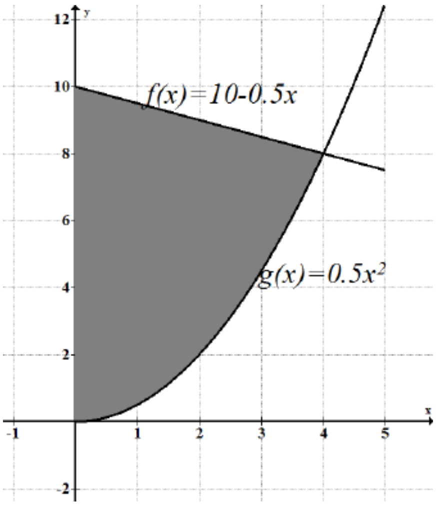
(1 mark)

(e) A machine part can be modelled by the shaded region bounded by the functions $f(x) = 10 - 0.5x$ and $g(x) = 0.5x^2$ and the y-axis as shown in the diagram below.



Determine the area of the shaded region.

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(3 marks)

(f) An epidemic spreads across a parish at the rate of $s'(t) = 6e^{0.2t}$ new cases per day, where t is the number of days since the epidemic began.

Determine the

(i) formula for the spread of the epidemic, $s(t)$

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(2 marks)

(iii) number of days it will take to reach 150 new cases.

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(2 marks)

Total 25 marks

END OF TEST

CARIBBEAN EXAMINATIONS COUNCIL

CARIBBEAN ADVANCED PROFICIENCY EXAMINATION®

INTEGRATED MATHEMATICS

PAPER 02

KEY AND MARK SCHEME

MAY/JUNE 2015

SPECIMEN PAPER

INTEGRATED MATHEMATICS

PAPER 02

KEY AND MARK SCHEME

Questions			Solutions	CK	AK	R	Total
1	(a)		$(x+y)+(x-y)i=7+3i$ Equating real parts $x+y=7$ Equation 1 Equating imaginary parts $x-y=3$ Equation 2		**1**		
			$Equation\ 1+Equation\ 2$ gives $x+x=7+3$ $2x=10$	**1**			
			$x=5$	**1**			**3**
			Hence, $x=5$ and $y=2$.				
	(b)	i)	$y=x$ and $y=4x-x^2$ At point of intersection, $x=4x-x^2$ $x^2+x-4x=0$			**1**	
			$x^2-3x=0$		**1**		
			$x(x-3)=0$ $x=0$ $x=3$ When $x=0$, in $y=x \Rightarrow y=0$ Therefore, $P(0,0)$ is a point of intersection				
			When $x=3$, in $y=x \Rightarrow y=3$	**1**			
			Therefore, $Q(3,3)$ is a point of intersection				**3**

INTEGRATED MATHEMATICS

PAPER 02

KEY AND MARK SCHEME

Questions			**Solutions**	**CK**	**AK**	**R**	**Total**
		ii)	Let M be the midpoint of P(0, 0) and Q(3, 3). Coordinates of M = $\left(\frac{0+3}{2},\frac{0+3}{2}\right)=\left(\frac{3}{2},\frac{3}{2}\right)$	1		1	
			Let m be the gradient of line segment PQ $m=\frac{3-0}{3-0}=1$ Let m_1 be the gradient of the perpendicular to the line PQ, $m\times m_1=-1$ $\Rightarrow m_1=-1$	1			4
			The equation of the perpendicular bisector passing through *M* with gradient m_1 is $y=mx+c$ $\frac{3}{2}=-1\left(\frac{3}{2}\right)+c$ $\Rightarrow c=3$ $\therefore equation\_is: y=-x+3$		1		
	(c)	i)	$(2+x)^3=\binom{3}{0}(2)^3(x)^0+\binom{3}{1}(2)^2(x)^1+\binom{3}{2}(2)^1(x)^2+\binom{3}{3}(2)^0(x)^3$		1		
			$(2+x)^3=(1)(8)(1)+(3)(4)(x)+(3)(2)(x^2)+(1)(1)(x^3)$ $(2+x)^3=8+12x+6x^2+x^3$		1		
			Replacing x with (- 3x) in solution from (i) above		1		3

INTEGRATED MATHEMATICS

PAPER 02

KEY AND MARK SCHEME

Questions			Solutions	CK	AK	R	Total
		ii)	$(2-3x)^3 = 8+12(-3x)+6(-3x)^2+(-3x)^3$ $(2-3x)^3 = 8-36x+6(9x^2)+(-27x^3)$ $(2-3x)^3 = 8-36x+54x^2-27x^3$		1 1		2
		iii)	$(2+x)^3-(2-3x)^3 = (8+12x+6x^2+x^3)-(8-36x+54x^2-27x^3)$ $(2+x)^3-(2-3x)^3 = 8+12x+6x^2+x^3-8+36x-54x^2+27x^3$ $(2+x)^3-(2-3x)^3 = 48x-48x^2+28x^3$		1	1	2
	(d)		$2x^2+x-15 \geq 0$ $\Rightarrow (2x-5)(x+3) \geq 0$ Case#1: $2x-5 \geq 0$ and $x+3 \geq 0$ $x \geq \frac{5}{2}$ and $x \geq -3$ $\Rightarrow x \geq \frac{5}{2}$ Case #2: $2x-5 \leq 0$ and $x+3 \leq 0$ $x \leq \frac{5}{2}$ and $x \leq -3$ $\Rightarrow x \leq -3$ The solution is $x \leq -3$ or $x \geq \frac{5}{2}$	1	1 1		3
	(e)		$\frac{\pi}{6}-\sin\left(\frac{\pi}{6}\right) = \frac{\pi}{6}-\frac{1}{2} = 0.0236$ If $cos\left(\theta+\frac{3\pi}{4}\right) = 0.5$, find θ in the range $-\pi \leq \theta \leq \pi$.		1	1	2

INTEGRATED MATHEMATICS

PAPER 02

KEY AND MARK SCHEME

Questions			Solutions	CK	AK	R	Total
	(f)		$\cos\left(\theta+\frac{3\pi}{4}\right)=0.5$ $\theta+\frac{3\pi}{4}=\cos^{-1}(0.5)$ $\theta+\frac{3\pi}{4}=\frac{\pi}{3},\frac{5\pi}{3}$ $\theta=\frac{\pi}{3}-\frac{3\pi}{4},\frac{5\pi}{3}-\frac{3\pi}{4}$ $\theta=-\frac{5\pi}{12},\frac{11\pi}{12}$	1	1	1	3
2.		i)	**Specific Objective Module 1.3.2, 1.3.4, 1.6.1, 1.8.2**	7	13	5	25
	(a)		$3^{x-1}\times 27^{3x-1}=81^{2x}$ $3^{x-1}\times 3^{3(3x-1)}=3^{4(2x)}$ $3^{x-1}\times 3^{9x-3}=3^{8x}$ $3^{x-1+9x-3}=3^{8x}$ $3^{10x-4}=3^{8x}$ $10x-4=8x$ $2x=4$ $x=2$		1 1	1	3
		ii)	$\log(x)+\log(x-3)=1$ $\log[(x)(x-3)]=1$ $\log(x^2-3x)=1$ $x^2-3x=10^1$ $x^2-3x-10=0$ $(x-5)(x+2)=0$ $x-5=0 \Rightarrow x=5$ $x+2=0 \Rightarrow x=-2(not\ possible)$	1	1 1	1	4

INTEGRATED MATHEMATICS

PAPER 02

KEY AND MARK SCHEME

Questions			Solutions	CK	AK	R	Total
	(b)	i)	$f(x)=x^3+bx^2-4x-5$				
			$\therefore f(3)=-8$	1			
			$(3)^3+b(3)^2-4(3)-5=-8$				3
			$27+9b-12-5=-8$		1		
			$9b+10=-8$	1			
			$9b=-18$				
			$b=-2$				
		(ii)			1		
			$g(x)=2x^3+7x^2+x-10$	1			
			$g(1)=2(1)^3+7(1)^2+(1)-10=0$		1		
			$\therefore g(1)=0$		1		4
			⇒x − 1 is a factor				
			$\begin{array}{r} 2x^2+9x+10 \\ x-1\overline{)2x^3+7x^2+x-10} \end{array}$				
			$\Rightarrow 2x^3+7x^2+x-10\equiv(x-1)(2x^2+9x+10)\equiv(x-1)(x+2)(2x+5)$				
	(c)			1			
			8th term of AP = 150 $\Rightarrow a+7d=150$		1	1	4
			53rd term of AP = −30 $\Rightarrow a+52d=-30$			1	
			Solve simultaneously *a* = 178 and *d* = −4				
	(d)						
		(i)	$\begin{bmatrix}1 & 1 & 1\\ 2 & -3 & 0\\ 4 & 1 & -2\end{bmatrix}\begin{bmatrix}x\\ y\\ z\end{bmatrix}=\begin{bmatrix}4\\ 7\\ 1\end{bmatrix}$	1	1		2
		(ii)	Note that				
			$\lvert A\rvert=\begin{vmatrix}1 & 1 & 1\\ 2 & -3 & 0\\ 4 & 1 & -2\end{vmatrix}=(1)(2--12)+(-2)(-3-2)=14+10=24$		1		
			$A_x=\begin{vmatrix}4 & 1 & 1\\ 7 & -3 & 0\\ 1 & 1 & -2\end{vmatrix}=(1)(7--3)+(-2)(-12-7)=10+38=48$		1		

INTEGRATED MATHEMATICS

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KEY AND MARK SCHEME

Questions			Solutions	CK	AK	R	Total
			$A_y = \begin{vmatrix} 1 & 4 & 1 \\ 2 & 7 & 0 \\ 4 & 1 & -2 \end{vmatrix} = (1)(2-28)+(-2)(7-8) = -26+2 = -24$		1	1	
			$A_z = \begin{vmatrix} 1 & 1 & 4 \\ 2 & -3 & 7 \\ 4 & 1 & 1 \end{vmatrix} = (1)(-3-7)-(1)(2-28)+(4)(2--12) = -10+26+56 = 72$				5
			$x = \frac{A_x}{A} = \frac{48}{24} = 2$ $y = \frac{A_y}{A} = \frac{-24}{24} = -1$ $z = \frac{A_z}{A} = \frac{72}{24} = 3$	1			
			Or alternative solution				
			Specific Objective Module 1.4.6, 1.7.2, 1.7.4	7	13	5	25

INTEGRATED MATHEMATICS

PAPER 02

KEY AND MARK SCHEME

Solution MO 2 Item 3

Question 3			**Solution**	**CK**	**AK**	**R**	**Total**
	(a)		Number of junk food items consumed per day, calorie count per day, money spent on junk food per day.	1			1
	(b)		1. The two groups were not randomly selected. 2. This sample of students will not be representative of the student population, since every student in year 1 and year 2 did not have an equal chance of being selected Other statistically sound reasons	1 2			3
	(c)		Randomly select students – Process 1: order student numbers in sequential order and number them from 1 to 300. Using a random number generator, provide 30 numbers between 1 and 300 inclusive, 15 for each year group. Process 2: Use Stratified Random Sampling for each year and again each program of each year.		2	1	3
	(d)		A bar chart – Both groups can be represented and easily compared.	1		1	2
	(e)		$\bar{x} = \frac{\sum x}{n} = \frac{2415}{15}$ $= 161$ $\sigma = \sqrt{\frac{\sum x^2}{n} - \bar{x}^2} = \sqrt{\frac{390525}{15} - 25921}$ $= \sqrt{114} = 10.7$		1 1 2 1		5

INTEGRATED MATHEMATICS

PAPER 02

KEY AND MARK SCHEME

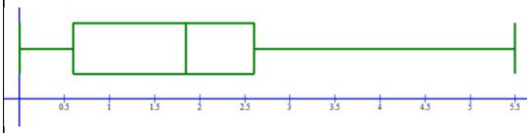
Question 3	**Solution**			**CR**	AK	R	TOT
	(f)		Year 1 masses are more homogenous. Reason: Standard deviation is less than that of year 2 weights. Standard deviation is a measure of spread Large standard deviation indicates non-homogeneity	1	1	1	3
	(g)	i)	$P(year1 + year2 \geq 170)$ $= \frac{\text{num 1} + \text{num 2}}{\text{total 1} + \text{total 2}}$ $= \frac{4+1}{15+15}$ $= \frac{5}{30} = \frac{1}{6}.$		1 1 1		3
		ii)	Amount of groups = C_5^{30} $= \frac{30!}{5!\,25!}$ $= 142506$	1	1 1		3
	(h)		Introduce the mandatory nutrition class. The mean of the first year group is larger. The variance of the first year group is smaller. These indicate that the Students of year 1 have more consistently higher masses than year two.			1 1	2
			Specific Objective Module 2.2.2, 2.3.2, 2.4.3, 2.5.2	**7**	**13**	**5**	**25**

INTEGRATED MATHEMATICS

PAPER 02

KEY AND MARK SCHEME

Solutions MO 2 Item 4

Question 4			Solutions	CK	AK	R	Total
4	(a)	(i)	$x+3x=48,$ $x=12$ $12\% \times 750=\frac{12}{100}\times 750\approx 90$		1 1	1	3
	(b)	i)	$0.0, 0.25, 0.4, 0.5, 0.6, 1.25, 1.4, 1.7, 1.8, 1.9, 2.0, 2.0, 2.3, 2.6, 3.0, 3.1, 3.5, 5.5$ $Median=\frac{1.8+1.9}{2}=1.85$ $Q_1=0.6$ $Q_3=2.6$	1 1	1		3
		ii)	 Axis Med & quart whiskers		1 1 1		3
	(c)	i)	$np=3.6$ then $npq=2.16$ $(np)q=3.6q=2.16$ $q=\frac{2.16}{3.6}=0.6$ $p=1-q=1-0.6=0.4$ $np=n(0.4)=3.6$ $n=\frac{3.6}{0.4}=9$ Alternatively in reverse.	1	1 1		3
		ii)	$P(X\geq 2)=1-P(X\leq 1)$ $=1-[P(X=0)+P(X=1)]$ $=1-[0.00218+0.0174]$ $=0.9804$	1	1	1	3
	(d)		$P(X\geq 32)=P\left(Z\geq \frac{32-21}{5}\right)$ $=P(Z\geq 2.2)$ Correct use of the normal table to obtain $P(X\geq 32)=P(Z\geq 2.2)=0.0139$	1 1	1 1		4
	(e)	i)	Use of equation $\hat{y}=5+0.4(x)$ $\hat{y}=5+0.4(15)=5+6=11$ estimated minutes late is 11		1 1	1	3
		ii)	r is small. Small r suggest weak correlation. So the regression line will give poor estimates.	1		1 1	3
			Specific Objective Module 2.2.2, 2.5.6, 2.5.7	7	13	5	25

INTEGRATED MATHEMATICS

PAPER 02

KEY AND MARK SCHEME

Solutions MO 3 Question 5

Question 5			Solutions	**CK**	**AK**	**R**	**Total**
5	(a)	i)	$2-1=1$		**1**		
		ii)	$3-2(2)=3-4=-1$		**1**		
		iii)	$f(x)$ is not continuous (i.e. discontinuous) at $x=2$ $\lim_{x\to 2^-} f(x) \neq \lim_{x\to 2^+} f(x)$	**1**		**1**	**4**
	(b)		$\lim_{x\to 2}\frac{x-2}{x^2-x-2}=\frac{2-2}{4-2-2}=\frac{0}{0}$, Indeterminate!! $\lim_{x\to 2}\frac{x-2}{x^2-x-2}=\lim_{x\to 2}\frac{x-2}{(x-2)(x+1)}$ $\lim_{x\to 2}\frac{1}{x+1}$ $\frac{1}{2+1}=\frac{1}{3}$	**1**	**1**	**1**	**3**
	(c)	i)	$y=\frac{e^{3x}}{5x+\sin x}=\frac{u}{v}$ $u'=3e^{3x}$ $v'=5+\cos x$ Using the Quotient Rule $y'=\frac{3e^{3x}(5x+sinx)-e^{3x}(5+cosx)}{(5x+\sin x)^2}$	**1** **1**	**1** **1**		**4**
		ii)	$y=(4x^2-lnx)^{\frac{1}{2}}$ $y'=\frac{1}{2}(4x^2-lnx)^{-\frac{1}{2}}\left(8x-\frac{1}{x}\right)$	**1**	**1** **1**		**3**

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PAPER 02

KEY AND MARK SCHEME

Question 5			Solutions	CK	AK	R	Total
	(d)		$g(x) = x^3 - 6x^2 + 9x$				
			$g'(x) = 3x^2 - 12x + 9$		**1**		
			$g''(x) = 6x - 12$		**1**		
			For stationary points				
			$g'(x) = 3x^2 - 12x + 9 = 0$				
			$\Rightarrow x^2 - 4x + 3 = 0$				
			$(x-3)(x-1) = 0$				
			$x = 3, y = 0$				
			$x = 1, y = 4$				
			For nature of stationary points,		**1**		
			$g''(x) = 6x - 12$		**1**		
			$g''(3) > 0$ (3,0) is a minimum point				
			$g''(1) < 0$ (1,4) is a maximum				
						1	
						1	**6**
	(e)	i)	$V(r,h) = \pi r^2 h$				
			Treating h as a constant,				
			$\frac{\partial V}{\partial r} = h\left(\frac{d}{dr}\pi r^2\right) = 2\pi rh$	**1**			**2**
			When r = 14cm and h = 28cm,				
			$\frac{\partial V}{\partial r} = 2(14)(28)\pi = 784\pi cm^3$		**1**		
		ii)	When comparing similar cylindrical tanks of the same height,				
			For every centimeter increase/decrease in the radius when r = 14cm, the volume/capacity of the tank increases/decreases by 784π cm^3 or 0.784π litres accordingly.			**1**	**1**

INTEGRATED MATHEMATICS

PAPER 02

KEY AND MARK SCHEME

Question 5			Solutions	**CK**	**AK**	**R**	**Total**
	(f)	(i) (ii)	$R(x) = -0.008x^3 + 0.24x^2 + 0.6x$ $R'(x) = -0.024x^2 + 0.48x + 0.6$ $R'(8) = -0.024(64) + 0.48(8) + 0.6$ $= 2.90\ (million\ dollars)\ or\ \$2\,900\,000$	1	1		2
			Specific Objectives Module 3.1.1, 3.1.2, 3.1.3, 3.1.4, 3.1.5, 3.2.3, 3.2.10, 3.3.3, 3.3.1	**7**	**13**	**5**	**25**

INTEGRATED MATHEMATICS

PAPER 02

KEY AND MARK SCHEME

Question 6			Solutions	CK	AK	R	Total
	a	i)	Let $u = 2x - 4$ $du = 2\,dx$ $\int (2x-4)^7\ dx = \frac{1}{2}\int u^7 du$ $\frac{1}{2}\left(\frac{u^8}{8}\right) = c$ $= \frac{(2x-4)^8}{8x2} + c$ $= \frac{(2x-4)^8}{16} + c$	1	1	1	3
		ii)	$\int 12x^3 - 2x^2 - 5\,dx = 3x^4 - \frac{2}{3}x^3 - 5x + c$	1	1		2
	b		$\int_0^\pi (2\cos x + 3\,sin\,x)dx = 2\sin x - 3\cos x\vert_0^\pi$ $= (2\sin\pi - 3\cos\pi) - (2\sin 0 - 3\cos 0)$ $= 3 - (-3) = 6$	1	1	1	3

INTEGRATED MATHEMATICS

PAPER 02

KEY AND MARK SCHEME

Question 6			Solutions	CK	AK	R	Total
	(c)		$\frac{dP}{dt} = kP$				
			$\int \frac{dP}{P} = \int kdt$				
			$\ln P = kt + C$		1		
			$P(t) = Ae^{kt}$			1	
			Using $P(0) = 1400$	1		1	
			$Ae^0 = 1400$				
			$A = 1400$				
			$P(t) = 1400e^{kt}$		1		
			Using $P(10) = 2100$				
			$1400e^{10k} = 2100$				
			$e^{10k} = \frac{2100}{1400}$				
			$10k = ln\frac{3}{2}$		1		
			$k = \frac{1}{10}ln\frac{3}{2}$				
			hence $P(t) = 1400e^{\frac{1}{10}ln\frac{3}{2}t}$.				
							6
	(d)	(i)	$N'(x) = \frac{dN}{dx} = \frac{6}{x+0.5}$	1	1		
			$\int dN = \int \frac{6}{x+0.5}dx$				
			$N(x) = 6\ln(x+0.5) + C$	1			
			When $x = 0$, $N(x) = 16$				
			$\therefore 16 = 6\ln 0.5 + c$ and $c = 20.2$				3
			$N(x) = 6\ln(x+0.5) + 20.2$				

INTEGRATED MATHEMATICS

PAPER 02

KEY AND MARK SCHEME

Question 6			Solutions	**CK**	**AK**	**R**	**Total**
	(d)	ii)	$N(4) = 6\ln(4+0.5) + 20.2 = 29.2\ \mathit{million\ dollars}$			1	1
	(e)	i)	$Area = \int_0^4 [f(x) - g(x)]dx$ $= \int_0^4 \left(10 - \frac{x}{2} - \frac{x^2}{2}\right)dx = \left[10x - \frac{x^2}{4} - \frac{x^3}{6}\right]_0^4$ $= \left(40 - 4 - \frac{32}{3}\right) = \frac{76}{3}$	1	1	1	3
	f	i)	$s(t) = \int 6e^{0.2t}dt$ $= 30e^{0.2t} + c$ when $t = 0 \quad s(0) = 0$ $30e^{0.2t} + c = 0$ $c = -30$ $s(t) = 30e^{0.2t} - 30$		1 1		2
	(f)	ii)	$30e^{0.2t} - 30 = 150$ $30e^{0.2t} = 180$ $e^{0.2t} = 6$ $0.2t \ln e = \ln 6$ $t = \frac{\ln 6}{0.2}$ $t \approx 8.95\text{days}$ $t \approx 9\text{days}$		1 1		2
			Specific Objectives Module 2.2.2, 2.5.6, 2.5.7	**7**	**13**	**5**	**25**

SPEC 2015/02167032

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INTEGRATED MATHEMATICS

SPECIMEN PAPER

Paper 032

1 hour 30 minutes

READ THE FOLLOWING IINSTRUCTIONS CAREFULLY.

1. This paper consists of a case study.
2. Read the case and use the information to answer the questions.
3. All answers must be written in this booklet.
4. You are advised to take some time to read through the paper and plan your answers.
5. You may use silent electronic, non-programmable calculators to answer questions.

DO NOT TURN THIS PAGE UNTIL YOU ARE TOLD TO DO SO.

INSTRUCTION: Read the following case study and answer the questions that follow.

CASE STUDY

A researcher at St John's High School, Tobago, investigated the average amount of pocket change that each of 100 sixth form students possesses. The research methodology for this study was divided into five major stages.

Stage 1: A target population of sixth form students at St. John's High School, Tobago was identified for analysis.

Stage 2: The stratified sampling design that was adopted had two distinguishable levels of students. The first strata comprised lower sixth students while the second strata comprised of upper sixth students. The main characteristics of the randomly selected participants from the target population were summarized in a cross-tabulation table.

Stage 3: A questionnaire instrument was designed.

Stage 4: The data for each student were extracted from the questionnaires, coded and saved into a database.

Stage 5: Statistical methods were used to analyse the results for this study.

Data Coding for Research Question

1) Variable Name – Sex
Variable Label – Sex of Student
Measure – Nominal

VALUE	VALUE LABEL (Sex of Student)
01	Male
02	Female

2) Variable Name – Level
Variable Label – Level of Sixth Form Student
Measure – Nominal

VALUE	VALUE LABEL (Age of Student)
01	Lower Sixth Student
02	Upper Sixth Student

3) Variable Name – Pocket Change
Variable Label – Pocket Change of Sixth Form Student
Measure – Scale

Extracted data from the questionnaire are summarized in Table 1 below.

TABLE 1

Sex	**Level**	**Pk Change (TT$)**
1	2	110.00
1	1	85.00
2	1	56.00
2	2	20.00
2	2	23.00
1	1	45.00
2	2	60.00
1	2	43.00
2	1	54.00
2	2	67.00
1	1	87.00
1	2	95.00
2	1	100.00
2	2	120.00
1	2	87.00
2	2	90.00
1	1	56.00
2	1	35.00
2	1	24.00
1	1	36.00
2	2	51.00
2	1	75.00
1	2	69.00
1	2	49.00
2	2	42.00
1	1	58.00
1	2	58.00
1	1	92.00
1	2	72.00
1	1	70.00
1	2	80.00
1	2	60.00

GO ON TO THE NEXT PAGE

1. Write a suitable title for this research project.

…………………………………………………………………………………………………

…………………………………………………………………………………………………

(2 marks)

2. State THREE limitations of the research method.

…………………………………………………………………………………………………

…………………………………………………………………………………………………

…………………………………………………………………………………………………

(3 marks)

3. List the ‘Pocket Change’ observed for each of the female students.

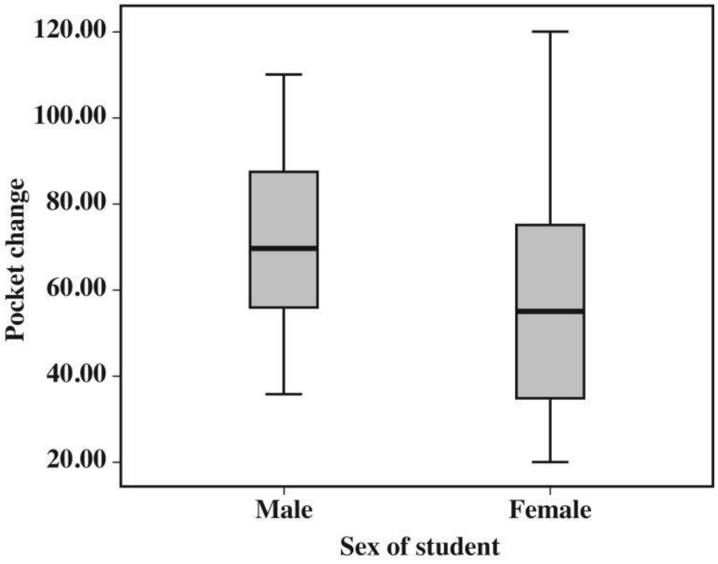
…………………………………………………………………………………………………

…………………………………………………………………………………………………

…………………………………………………………………………………………………

(3 marks)

4. Consider the box-and-whiskers plot below.



Hence or otherwise,

(a) calculate the maximum and minimum ‘Pocket Change’ for the female sixth form students

……………………………………………..…………..………………………………..…………

……………………………………………..…………..…………………………………………….

(2 marks)

(b) comment on the shape of the distribution for the female students.

…………………………………..……………………………………..……………………………………

……………………….……………………………………………………………………………………

(2 marks)

5. Calculate the sample mean for the ‘Pocket Change’ of the female students.

…………………………………………………………………………………………………

…………………………………………………………………………………………………

…………………………………………………………………………………………

(3 marks)

6. Calculate the standard deviation for the ‘Pocket Change’ of the female students.

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(5 marks)

7. Calculate the median ‘Pocket Change’ for the female students.

…………………………………………………………………………………………………

…………………………………………………………………………………………………

(2 marks)

8. Calculate the upper quartile ‘Pocket Change’ for the female students.

…………………………………………………………………………………………………

…………………………………………………………………………………………………

(2 marks)

9. Calculate the lower quartile ‘Pocket Change’ for the female students.

…………………………………………………………………………………………………

…………………………………………………………………………………………………

(2 marks)

GO ON TO THE NEXT PAGE

10. Construct a stem and leaf diagram that shows the amount of 'Pocket Change' for the male students.

……

……

……

……

(4 marks)

11. (a) Prepare a 2×2 cross–tabulation table (that is a 2×2 contingency table) that summarizes the data stratification process and complete Table 2 below.

TABLE 2

	Lower Sixth	**Upper Sixth**	**Total**
Male			
Female			
Total			

…………………………………………………………………………………………………

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(9 marks)

(b) Using the results from (a), find the probability that a sixth form student from St John's High School is

(i) in the Upper Sixth form

…………………………………………………………………………………………………

…………………………………………………………………………………………………

…………………………………………………………………………………………………

(3 marks)

GO ON TO THE NEXT PAGE

(ii) an Upper Sixth form female

……………………………………………………………………..……..………………………

……………………………………………………………..………..……………………

………………………………………………………………………………………………

(3 marks)

(iii) a Lower Sixth form student, given that the student is female

………………………………………………………………………………………………

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…………………………………………………………………..……………………………

(4 marks)

(iv) a female or Lower Sixth form student.

………………………………………………………………………………………………

…………………………………………………………………..……………………………

…………………………………………………………………..……………………………

…………………………………………………………………..……………………………

(4 marks)

12. Design a questionnaire instrument that can be used to collect the summarized information in Table 1.

……………………………………………………………………………………………………

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(7 marks)

Total 60 marks

END OF TEST

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INTEGRATED MATHEMATICS

PAPER 032

CASE STUDY

KEY AND MARK SCHEME

MAY/JUNE 2015

SPECIMEN PAPER

INTEGRATED MATHEMATICS

CASE STUDY - PAPER 032

KEY AND MARK SCHEME

	CK	AK	R
SOLUTIONS			
1. Project title for this research project **[2 marks]**			
Solution			
An Investigation of the amount of pocket change in the possession of Sixth Form Students at St. John’s High School, Tobago.		1	1
2. Limitations of the research methodology. **[3 marks]**			
Solution			
First, the target population was Sixth Form Students at St. John’s High School in Tobago and was assumed that all Sixth Form students had pocket change.			1
Second, the sampling method could have been optimized.			1
Third, the results of this study may not be fully generalizable for the entire population of Sixth Form students at St. John’s High School.			1

INTEGRATED MATHEMATICS

CASE STUDY - PAPER 032

KEY AND MARK SCHEME

<table>
<tr><th></th><th>CK</th><th>AK</th><th>R</th></tr>
<tr><td>

3. 'Pocket Change' observed for each of the female students.

[3 marks]

Solution

<table>
<tr><td>Sex</td><td>PkChange($)</td></tr>
<tr><td>2</td><td>56.00</td></tr>
<tr><td>2</td><td>20.00</td></tr>
<tr><td>2</td><td>23.00</td></tr>
<tr><td>2</td><td>60.00</td></tr>
<tr><td>2</td><td>54.00</td></tr>
<tr><td>2</td><td>67.00</td></tr>
<tr><td>2</td><td>100.00</td></tr>
<tr><td>2</td><td>120.00</td></tr>
<tr><td>2</td><td>90.00</td></tr>
<tr><td>2</td><td>35.00</td></tr>
<tr><td>2</td><td>24.00</td></tr>
<tr><td>2</td><td>51.00</td></tr>
<tr><td>2</td><td>75.00</td></tr>
<tr><td>2</td><td>42.00</td></tr>
</table>

</td><td></td><td>1</td><td>1
1</td></tr>
</table>

	CK	**AK**	**R**
4. Box-and-whiskers plot.			
Solution			
(a) The maximum pocket change for the female Six Form students is \$120. The minimum pocket change for the female Six Form students is \$20.	**1**		**1**
(b) The distribution for the female students appears to be symmetrical.	**1**		**1**
5. Calculate the sample mean for the 'Pocket Change' of the female students. [3 marks]			
Solution			
Let $\bar{x}$ denote the mean 'Pocket Change' of the female students.			
$\bar{x} = \frac{\sum x}{n}$ $= \frac{56+20+23+60+54+67+100+120+90+35+24+51+75+42}{14}$	**1**		
$= \frac{817}{14}$		**1**	
$= 58.3571$ $= \$58.36$	**1**		

INTEGRATED MATHEMATICS

CASE STUDY - PAPER 032

KEY AND MARK SCHEME

<table>
<tr><th></th><th>CK</th><th>AK</th><th>R</th></tr>
<tr><td>
6. Standard deviation ‘Pocket Change’ of the female students. **[3 marks]**

Solution

Consider the table below

<table>
<tr><th>x_i</th><th>$\bar{x}$</th><th>$x_i - \bar{x}$</th><th>$(x_i - \bar{x})^2$</th></tr>
<tr><td>56</td><td>58.35714</td><td>-2.35714</td><td>5.556109</td></tr>
<tr><td>20</td><td>58.35714</td><td>-38.3571</td><td>1471.27</td></tr>
<tr><td>23</td><td>58.35714</td><td>-35.3571</td><td>1250.127</td></tr>
<tr><td>60</td><td>58.35714</td><td>1.64286</td><td>2.698989</td></tr>
<tr><td>54</td><td>58.35714</td><td>-4.35714</td><td>18.98467</td></tr>
<tr><td>67</td><td>58.35714</td><td>8.64286</td><td>74.69903</td></tr>
<tr><td>100</td><td>58.35714</td><td>41.64286</td><td>1734.128</td></tr>
<tr><td>120</td><td>58.35714</td><td>61.64286</td><td>3799.842</td></tr>
<tr><td>90</td><td>58.35714</td><td>31.64286</td><td>1001.271</td></tr>
<tr><td>35</td><td>58.35714</td><td>-23.3571</td><td>545.556</td></tr>
<tr><td>24</td><td>58.35714</td><td>-34.3571</td><td>1180.413</td></tr>
<tr><td>51</td><td>58.35714</td><td>-7.35714</td><td>54.12751</td></tr>
<tr><td>75</td><td>58.35714</td><td>16.64286</td><td>276.9848</td></tr>
<tr><td>42</td><td>58.35714</td><td>-16.3571</td><td>267.556</td></tr>
<tr><td>$\sum x_i = 817$</td><td></td><td></td><td>$\sum (x_i - \bar{x})^2 = 11683.21$</td></tr>
</table>
Let s denote the standard deviation of the ‘Pocket Change’ of the female students.
</td><td></td><td>1</td><td></td></tr>
</table>

INTEGRATED MATHEMATICS

CASE STUDY - PAPER 032

KEY AND MARK SCHEME

	CK	AK	R
Now,		1	
$$s^2 = \frac{\sum_{i=1}^{14}(x-\bar{x})^2}{n-1}$$		1	
From table above, $\sum_{i=1}^{14}(x_i-\bar{x})^2 = 11683.21$, $n = 14$			
So,			
$$s^2 = \frac{11683.21}{14-1}$$			1
$s^2 = 898.7088$ $s = 29.97847$ $s = \$29.98$	1		

	CK	AK	R
7. Calculate the median for the amount of pocket change for the female students. **[3 marks]**			
Solution			
Consider the amount of pocket change for each of the female students.			
56, 20, 23, 60, 54, 67, 100, 120, 90, 35, 24, 51, 75, 42			
Arranging the data in ascending order gives			
20, 23, 24, 35, 42, 51, 54, 56, 60, 67, 75, 90, 100, 120			1
There are 14 data points.		1	
The median is the average of the 7^{th} and 8^{th} data points			
$Median = \frac{54+56}{2}$			
$Median = \frac{110}{2}$			
$\therefore Median = \$55$			

INTEGRATED MATHEMATICS

CASE STUDY - PAPER 032

KEY AND MARK SCHEME

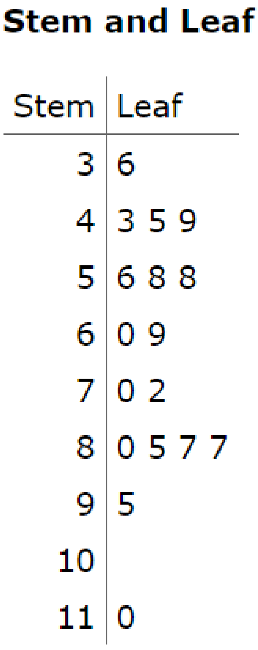
	CK	AK	R
8. The upper quartile 'Pocket Change' for the female students **[3 marks]**			
Solution			
Recall, the 'Pocket Change' for the female Six Form students when summarized in ascending order is as follows			
20, 23, 24, 35, 42, 51, 54, 56, 60, 67, 75, 90, 100, 120			**1**
The upper section of the data from the 8^{th} data point to the 14^{th} data point is	**1**		
56, 60, 67, 75, 90, 100, 120			
The Upper Quartile is 75.			

	CK	**AK**	**R**
9. Calculate the lower quartile for the 'Pocket Change' for the female students. **[3 marks]**			
Solution			
Recall, the 'Pocket Change' for the female Six Form students when summarized in ascending order is as follows			
20, 23, 24, 35, 42, 51, 54, 56, 60, 67, 75, 90, 100, 120			
The lower section of the data from the 1^{st} data point to the 7^{th} data point is 20, 23, 24, 35, 42, 51, 54			1
The Lower Quartile is 35.	1		

INTEGRATED MATHEMATICS

CASE STUDY - PAPER 032

KEY AND MARK SCHEME

<table>
<tr><th></th><th>CK</th><th>AK</th><th>R</th></tr>
<tr><td>10. Stem and leaf diagram that shows the amount of pocket change for the female students [3 marks]</td><td></td><td></td><td></td></tr>
<tr><td><u>Solution</u></td><td></td><td></td><td></td></tr>
<tr><td>From, table 1, the amount of pocket change for the male Sixth Form students is as follows</td><td></td><td></td><td></td></tr>
<tr><td>110, 85, 45, 43, 87, 95, 87, 56, 36, 69, 49, 58, 58, 92, 72, 70, 80, 60</td><td></td><td></td><td></td></tr>
<tr><td>Rearranging the amount of pocket change in ascending order gives</td><td></td><td></td><td></td></tr>
<tr><td>36, 43, 45, 49, 56, 58, 58, 60, 69, 70, 72, 80, 85, 87, 87, 95, 110</td><td></td><td></td><td></td></tr>
<tr><td>Stem-and-Diagram for “The amount of pocket change for male Sixth Form Students”</td><td></td><td></td><td>1</td></tr>
<tr><td>Key: 3|6 = 36

Stem and Leaf Plot:
<table>
<tr><th>Stem</th><th>Leaf</th></tr>
<tr><td>3</td><td>6</td></tr>
<tr><td>4</td><td>3 5 9</td></tr>
<tr><td>5</td><td>6 8 8</td></tr>
<tr><td>6</td><td>0 9</td></tr>
<tr><td>7</td><td>0 2</td></tr>
<tr><td>8</td><td>0 5 7 7</td></tr>
<tr><td>9</td><td>5</td></tr>
<tr><td>10</td><td></td></tr>
<tr><td>11</td><td>0</td></tr>
</table></td><td>1</td><td>1</td><td>1</td></tr>
</table>

<table>
<tr><th></th><th>CK</th><th>AK</th><th>R</th></tr>
<tr><td>

11.(a) 2×2 cross tabulation table (i.e. a 2×2 contingency table) that summarizes the data stratification process and complete Table 2 below.

Solution

Table 2

	LOWER SIX	UPPER SIX	TOTAL
MALE	8	10	18
FEMALE	6	8	14
TOTAL	14	18	32

</td><td>1
1
1</td><td>1
1
1</td><td>1
1
1</td></tr>
<tr><td>

11. (b) i) upper sixth form **[3 marks]**

Solution

$$p(upper\ six\ student\ )$$

$$= \frac{\text{Total number of upper six students}}{Total\ number\ of\ six\ form\ students}$$

$$= \frac{18}{32}$$

</td><td>1</td><td>1
1</td><td></td></tr>
<tr><td>

11.(b) ii) an upper sixth form female **[3 marks]**

Solution

$$p(upper\ six\ female)$$

$$= \frac{\text{Total number of upper six female}}{Total\ number\ of\ six\ form\ students}$$

$$= \frac{8}{32}$$

</td><td>1</td><td>1
1</td><td></td></tr>
</table>

	CK	AK	R
11. (b) iii) Lower Sixth Form **[4 marks]**			
Solution			
$p\left(lower\ six\ student\ given\ that\ student\ is\ female\right)$		1	
$= p(lower\ six\ student \mid the\ student\ is\ female)$		1	
$= \dfrac{\text{Total number of female lower six students}}{Total\ number\ of\ female\ six\ form\ students}$		1	
$= \dfrac{6}{14}$			1
Find the probability that a sixth form student from St John's High school is			
[4 marks]			
11. (b) iv) a female or lower six student			
Solution			
$p(female\ or\ lower\ six\ student)$ $= p(female) + p(lower\ six\ student) - p(female \cap lower\ six\ student)$		1	
$p(female) = \dfrac{14}{32}$, $p(lower\ six\ student) = \dfrac{14}{32}$,			
$p\left(female \cap lower\ six\ student\right)$ $= p(female) \times p(lower\ six\ given\ that\ student\ is\ female)$ $= \dfrac{14}{32} \times \dfrac{6}{14}$, $= \dfrac{6}{32}$,	1		
$p(female\ or\ lower\ six\ student)$ $= \dfrac{14}{32} + \dfrac{14}{32} - \dfrac{6}{32} = \dfrac{22}{32}$		1	1

INTEGRATED MATHEMATICS

CASE STUDY - PAPER 032

KEY AND MARK SCHEME

	CK	AK	R
13. Design a questionnaire instrument that can collect the summarized information in Table 1. **[7 marks]**			
Solution QUESTIONNAIRE This questionnaire investigates the amount of pocket change that each Sixth Form student possesses. All responses will be recorded and treated with strict confidence. Your answers are important to the success of this study and we thank you for your assistance. Please tick your choice for the appropriate questions or where necessary fill in the blanks.			
1) What is your sex?			**1**
( ) Male			**1**
( ) Female			
2) What is your academic level?	**1**		
( ) Lower Six Student			
( ) Upper Six Student			
3) Your pocket change is ___________ dollars.	**1**		**1** **1** **1**
	16	**21**	**23**